國立臺北大學 109 學年度碩士班一般入學考試試題

系（所）組別：金融與合作經營學系

科　　目：統計學

☑可 ☐不可使用計算機

1. (10%) Suppose X_1, X_2 are independent and both follow continuous uniform distribution U(0,1). Please find the probability $Pr(X_1^2+X_2^2<1)$.

2. (15%) Let X equal the weight in grams of a miniature candy bar. Assume that $\mu = E(X) = 24.43$ and $\sigma^2 = Var(X) = 1.875$. Let $\bar{X}=\sum_{i=1}^{n} X_i / n$ be the sample mean of a random sample of n = 30 candy bars. Find

   (1) (5%) $E(\bar{X})$

   (2) (5%) $Var(\bar{X})$

   (3) (5%) $Pr(24.18 \leq \bar{X} \leq 24.93)$ (Hint: Use the empirical rule for normal distributions)

3. (10%) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the distribution with p.d.f. $f(x, \theta) = \theta x^{\theta-1}$, $0 < x < 1$, $0 < \theta < \infty$. Assume that $\bar{X}=\sum_{i=1}^{n} X_i / n = 0.2$ be the sample mean. Please use $\bar{X}$ to find the moments estimator of θ and calculate its estimate.

4. (15%) Let $Y_1, Y_2, \ldots, Y_{11}$ be a random sample of size n = 11 from Group 1 with mean μ_1 and variance σ_1^2; and their sample mean is $\bar{\mu}_1$ and sample variance is $S_1^2 = \left(\sum_{i=1}^{11}(Y_i-\bar{\mu}_1)\right)\Big/(n-1)$. Let $Y_{12}, Y_{13}, \ldots, Y_{24}$ be the other random sample of size m = 13 from Group 2 with population mean μ_2 and variance σ_2^2; and their sample mean is $\bar{\mu}_2$ and sample variance is $S_2^2 = \left(\sum_{i=12}^{24}(Y_i-\bar{\mu}_2)\right)\Big/(m-1)$. Let D_i be a dummy, whose value is 1 if Y_i from Group 1, and is 0 if Y_i from Group 2, i = 1, 2, …, 24. A simple regression is estimated as follows:

$$\hat{Y}_i = 1.66 - 0.63D_i,$$

   whose SSE = $\sum_{i=1}^{24}\left(Y_i-\hat{Y}_i\right) = 6.6$.

   (1) (5%) What is the value of $(\bar{\mu}_2 - \bar{\mu}_1)$?

   (2) (10%) Assume $\sigma_1^2 = \sigma_2^2$, please find an estimate of $Var(\bar{\mu}_2 - \bar{\mu}_1)$. (Hint: $SSE = (n-1)S_1^2 + (m-1)S_2^2$)

5. (10%) The probabilities of two independent events A and B：P(A)=0.2, P(B)=0.5. Please calculate $P(A^c \cap B^c)$. A^c and B^c represent the complement of A and B.

6. (10%) Let $X_1, X_2 \ldots X_n$ be i.i.d. exponential random variables with the p.d.f. $f(x|\beta) = \beta e^{-\beta x}$. Please find the maximum likelihood estimator of β.

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7. (15%) Hazard function is usually employed in the study of survival analysis. The hazard function of a positive random variable X is defined as:

$$\lambda(t)=\frac{f(t)}{1-F(t)}\text{，}t>0$$

f(t) and F(t) represent the p.d.f. and c.d.f of X respectively. Generally, defining S(t)=1-F(t) as survival function being the probability lasts longer than t time units.

(1) (5%)Explain the meaning of "hazard function".

(2) (10%)If $\lambda(t)$ is a constant, that is $\lambda(t)\equiv\lambda>0$, please depict the distribution of X.

8. (15%) A zoologist observes the Polar bears activities nearby the North Pole for 50 weeks. He records the number of bears he finds every week. Please test if the number of bears follows Poisson distribution under significant level $\alpha=0.05$?

Number of Observations (Y)	Frequency (Number of weeks)
0	32
1	12
2	6
3 or Above	0

附表一、卡方(x^2)分布百分數值表。表中，ν為自由度。

Percentage Points of the x^2 Distribution[a]

	α								
ν	.995	.990	.975	.950	.500	.050	.025	.010	.005
1	0.00 +	0.00 +	0.00 +	0.00 +	0.45	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	1.39	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	2.37	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	3.36	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	4.35	11.07	12.38	15.09	16.75
6	0.68	0.87	1.24	1.64	5.35	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	6.35	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	7.34	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	8.34	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	9.34	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	10.34	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	11.34	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	12.34	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	13.34	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	14.34	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	15.34	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	16.34	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	17.34	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	18.34	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	19.34	31.41	34.17	37.57	40.00
25	10.52	11.52	13.12	14.61	24.34	37.65	40.65	44.31	46.93
30	13.79	14.95	16.79	18.49	29.34	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	39.34	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	49.33	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	59.33	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	69.33	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	79.33	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	89.33	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	99.33	124.34	129.56	135.81	140.17

試題隨卷繳交