

國立政治大學 109 學年度 碩士暨碩士在職專班 招生考試試題

第 1 頁，共 4 頁

考 試 科 目	微積分	系 所 別	國際經營與貿易學系國際 經濟、國際財管、國際企 管與行銷組一般生	考 試 時 間	2 月 7 日(五) 第四 節
<p>Multiple choice questions (5 points each). 選擇題請在答案卡上作答，否則不予計分。</p> <p>1. Let <math>f(x) = x^4 - 2x^2 + 1</math> for <math>x &gt; -1</math>. Which of the following statements is true?</p> <p>(a) <math>f'(0) \leq 1</math> and <math>f''(0) \leq 1</math>.  (b) <math>f'(0) \leq 1</math> and <math>f''(0) &gt; 1</math>.  (c) <math>f'(0) &gt; 1</math> and <math>f''(0) \leq 1</math>.  (d) <math>f'(0) &gt; 1</math> and <math>f''(0) &gt; 1</math>.  (e) <math>f'(0)</math> does not exist.</p> <p>2. Let <math>f(x) = (1+x) \ln(1+x)</math> for <math>x &gt; -1</math>. Which of the following statements is true?</p> <p>(a) <math>f'(0) \leq 1</math> and <math>f''(0) \leq 1</math>.  (b) <math>f'(0) \leq 1</math> and <math>f''(0) &gt; 1</math>.  (c) <math>f'(0) &gt; 1</math> and <math>f''(0) \leq 1</math>.  (d) <math>f'(0) &gt; 1</math> and <math>f''(0) &gt; 1</math>.  (e) <math>f'(0)</math> does not exist.</p> <p>3. Suppose that <math>f</math> is a differentiable function on <math>(-\infty, \infty)</math> such that <math>f(1) = 1</math> and <math>f'(1) = 2</math>. Let</p> $h(x) = \begin{cases} (f(x) - 1)/(x - 1) & \text{if } x \neq 1; \\ 2 & \text{if } x = 1. \end{cases}$ <p>Which of the following statements can be concluded based on the given information?</p> <p>(a) <math>h'(1) \leq 1</math>.  (b) <math>1 &lt; h'(1) \leq 2</math>.  (c) <math>h'(1) &gt; 2</math>.  (d) <math>h'(1)</math> does not exist.  (e) None of the above statements can be concluded.</p> <p>4. Suppose that <math>f</math> is a differentiable function on <math>(-\infty, \infty)</math> such that <math>f(1) = 1</math> and <math>f'(1) = 2</math>. Let <math>h(x) = f(e^{2x-2})/x</math> for <math>x &gt; 0</math>. Which of the following statements can be concluded based on the given information?</p> <p>(a) <math>h'(1) \leq 1</math>.  (b) <math>1 &lt; h'(1) \leq 2</math>.  (c) <math>h'(1) &gt; 2</math>.  (d) <math>h'(1)</math> does not exist.  (e) None of the above statements can be concluded.</p> <p>5. Let <math>f(x) = x^2 + \sin(x)</math> and <math>g(x) = x^2 + \cos(x)</math>. Which of the following statements is true?</p> <p>(a) <math>3 &lt; \lim_{x \rightarrow \infty} g(x)/f(x) &lt; \infty</math>.  (b) <math>2 &lt; \lim_{x \rightarrow \infty} g(x)/f(x) \leq 3</math>.  (c) <math>1 &lt; \lim_{x \rightarrow \infty} g(x)/f(x) \leq 2</math>.  (d) <math>-\infty &lt; \lim_{x \rightarrow \infty} g(x)/f(x) \leq 1</math>.  (e) <math>\lim_{x \rightarrow \infty} g(x)/f(x)</math> does not exist.</p>					
備 註	<p>一、作答於試題上者，不予計分。選擇題請在答案卡上作答，否則不予計分。  二、試題請隨卷繳交。</p>				

國立政治大學 109 學年度 碩士暨碩士在職專班 招生考試試題

第 2 頁，共 4 頁

考 試 科 目 微積分	系 所 別	國際經營與貿易學系國際 經濟、國際財管、國際企 管與行銷組一般生	考 試 時 間	2 月 7 日(五) 第四 節
<p>6. Let <math>f(x) = (x - 3)^2</math> and <math>g(x) = \ln(x - 3)</math> for <math>x &gt; 3</math>. Which of the following statements is true?</p> <p>(a) <math>0 &lt; \lim_{x \rightarrow 3^+} f(x)g(x) &lt; \infty</math>.</p> <p>(b) <math>-1 &lt; \lim_{x \rightarrow 3^+} f(x)g(x) \leq 0</math>.</p> <p>(c) <math>-2 &lt; \lim_{x \rightarrow 3^+} f(x)g(x) \leq -1</math>.</p> <p>(d) <math>-\infty &lt; \lim_{x \rightarrow 3^+} f(x)g(x) \leq -2</math>.</p> <p>(e) None of the above statements holds true.</p> <p>7. Let <math>f(x) = \sqrt{x(2+x)}</math> and <math>g(x) = x</math> for <math>x &gt; 0</math>. Which of the following statements is true?</p> <p>(a) <math>-\infty &lt; \lim_{x \rightarrow \infty} (f(x) - g(x)) \leq 0</math>.</p> <p>(b) <math>0 &lt; \lim_{x \rightarrow \infty} (f(x) - g(x)) \leq 1</math>.</p> <p>(c) <math>1 &lt; \lim_{x \rightarrow \infty} (f(x) - g(x)) \leq 2</math>.</p> <p>(d) <math>2 &lt; \lim_{x \rightarrow \infty} (f(x) - g(x)) &lt; \infty</math>.</p> <p>(e) None of the above statements holds true.</p> <p>8. Suppose that <math>f'(x) = x^2(x+2)e^x</math> for <math>x \in (-\infty, \infty)</math>. Which of the following statements is true?</p> <p>(a) <math>f</math> is strictly increasing on the interval <math>(-1, \infty)</math>.</p> <p>(b) <math>f</math> is strictly decreasing on the interval <math>(0, \infty)</math>.</p> <p>(c) <math>f</math> has at least one local minimum on the interval <math>(-1, \infty)</math>.</p> <p>(d) <math>f</math> has at least one local maximum on the interval <math>(-3, \infty)</math>.</p> <p>(e) None of the above statements holds true.</p> <p>9. Let <math>f(x) = x - \int_0^x e^{t^2} dt</math> for <math>x \in (-\infty, \infty)</math>. Which of the following statements is true?</p> <p>(a) <math>f'(1) &gt; 0</math>.</p> <p>(b) <math>f</math> has a local maximum at 1.</p> <p>(c) <math>f</math> has a local minimum at 1.</p> <p>(d) <math>f'(0) \leq -2</math>.</p> <p>(e) None of the above statements holds true.</p> <p>10. Let <math>f(x) = x^2 - \int_0^x e^{t^2} dt</math> for <math>x \in (-\infty, \infty)</math>. Which of the following statements is true?</p> <p>(a) <math>0 &lt; \lim_{x \rightarrow 0} f(x)/x &lt; \infty</math>.</p> <p>(b) <math>-1 &lt; \lim_{x \rightarrow 0} f(x)/x \leq 0</math>.</p> <p>(c) <math>-2 &lt; \lim_{x \rightarrow 0} f(x)/x \leq -1</math>.</p> <p>(d) <math>-\infty &lt; \lim_{x \rightarrow 0} f(x)/x \leq -2</math>.</p> <p>(e) None of the above statements holds true.</p>				
備 註	<p>一、作答於試題上者，不予計分。選擇題請在答案卡上作答，否則不予計分。</p> <p>二、試題請隨卷繳交。</p>			

國立政治大學 109 學年度 碩士暨碩士在職專班 招生考試試題

第 3 頁，共 4 頁

考試科目	微積分	系所別	國際經營與貿易學系國際經濟、國際財管、國際企管與行銷組一般生	考試時間	2 月 7 日(五) 第四節
<p>11. Suppose that <math>f</math> is a twice differentiable function on <math>(-\infty, \infty)</math> such that <math>f(1) = 1</math>, <math>f(2) = 2</math> and <math>f(3) = -2</math>. Which of the following statements can be concluded based on the given information?</p> <p>(a) <math>f'(x) &gt; -3</math> for some <math>x</math> in the interval <math>(2, 3)</math>.</p> <p>(b) <math>f''(x) &lt; -2</math> for some <math>x</math> in the interval <math>(1, 3)</math>.</p> <p>(c) <math>f'(x) = -3</math> for some <math>x</math> in the interval <math>(1, 3)</math>.</p> <p>(d) <math>f'(x) &lt; -3</math> for some <math>x</math> in the interval <math>(1, 2)</math>.</p> <p>(e) None of the above statements can be concluded.</p> <p>12. Which of the following statements is true?</p> <p>(a) <math>\int_0^1 x^2 dx \leq 0.5</math> and <math>\int_0^\infty e^{-x/2} dx \leq 1</math>.</p> <p>(b) <math>\int_0^1 x^2 dx \leq 0.5</math> and <math>\int_0^\infty e^{-x/2} dx &gt; 1</math>.</p> <p>(c) <math>0.5 &lt; \int_0^1 x^2 dx \leq 2</math> and <math>\int_0^\infty e^{-x/2} dx \leq 1</math>.</p> <p>(d) <math>0.5 &lt; \int_0^1 x^2 dx \leq 2</math> and <math>\int_0^\infty e^{-x/2} dx &gt; 1</math>.</p> <p>(e) None of the above statements holds true.</p> <p>13. Which of the following statements is true?</p> <p>(a) <math>\int_0^\pi x \sin(x) dx \leq 0.5</math> and <math>\int_0^\infty x e^{-x/2} dx \leq 1</math>.</p> <p>(b) <math>\int_0^\pi x \sin(x) dx \leq 0.5</math> and <math>\int_0^\infty x e^{-x/2} dx &gt; 1</math>.</p> <p>(c) <math>0.5 &lt; \int_0^\pi x \sin(x) dx \leq 2</math> and <math>\int_0^\infty x e^{-x/2} dx \leq 1</math>.</p> <p>(d) <math>0.5 &lt; \int_0^\pi x \sin(x) dx \leq 2</math> and <math>\int_0^\infty x e^{-x/2} dx &gt; 1</math>.</p> <p>(e) None of the above statements holds true.</p> <p>14. Suppose that <math>\{a_n\}_{n=1}^\infty</math> is a sequence such that <math>a_1 = 1</math> and <math>a_{n+1} = \sqrt{0.5a_n + 0.06}</math> for <math>n \geq 1</math>. Which of the following statements is true?</p> <p>(a) <math>-\infty &lt; \lim_{n \rightarrow \infty} a_n \leq 0</math>.</p> <p>(b) <math>0 &lt; \lim_{n \rightarrow \infty} a_n \leq 0.5</math>.</p> <p>(c) <math>0.5 &lt; \lim_{n \rightarrow \infty} a_n \leq 1</math>.</p> <p>(d) <math>1 &lt; \lim_{n \rightarrow \infty} a_n &lt; \infty</math>.</p> <p>(e) None of the above statements holds true.</p> <p>15. Which of the following statements is true?</p> <p>(a) <math>\sum_{n=1}^\infty (n^2 + 1)/(2n^3 + n + 1) &lt; \infty</math> and <math>\sum_{n=1}^\infty (-1)^n/n</math> diverges.</p> <p>(b) <math>\sum_{n=1}^\infty (n^2 + 1)/(2n^3 + n + 1) &lt; \infty</math> and <math>\sum_{n=1}^\infty (-1)^n/n</math> converges conditionally.</p> <p>(c) <math>\sum_{n=1}^\infty (n^2 + 1)/(2n^3 + n + 1) = \infty</math> and <math>\sum_{n=1}^\infty (-1)^n/n</math> diverges.</p> <p>(d) <math>\sum_{n=1}^\infty (n^2 + 1)/(2n^3 + n + 1) = \infty</math> and <math>\sum_{n=1}^\infty (-1)^n/n</math> converges conditionally.</p> <p>(e) <math>\sum_{n=1}^\infty (-1)^n/n</math> converges absolutely.</p>					
備註	<p>一、作答於試題上者，不予計分。選擇題請在答案卡上作答，否則不予計分。</p> <p>二、試題請隨卷繳交。</p>				

國立政治大學 109 學年度 碩士暨碩士在職專班 招生考試試題

第 4 頁，共 4 頁

考 試 科 目	微積分	系 所 別	國際經營與貿易學系國際 經濟、國際財管、國際企 管與行銷組一般生	考 試 時 間	2 月 7 日(五) 第四 節
<p>16. Let <math>f(x, y) = 4xy - 2x + 2y</math> for <math>x, y \in (-\infty, \infty)</math>, and let <math>D_1 = \{(x, y) : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}</math>. Which of the following statements is true?</p> <p>(a) <math>3 &lt; \int_{D_1} f(x, y) d(x, y) &lt; \infty</math>.  (b) <math>2 &lt; \int_{D_1} f(x, y) d(x, y) \leq 3</math>.  (c) <math>1 &lt; \int_{D_1} f(x, y) d(x, y) \leq 2</math>.  (d) <math>0 &lt; \int_{D_1} f(x, y) d(x, y) \leq 1</math>.  (e) None of the above statements holds true.</p> <p>17. Let <math>f(x, y) = 4xy - 2x + 2y</math> for <math>x, y \in (-\infty, \infty)</math>, and let <math>D_1 = \{(x, y) : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq x\}</math>. Which of the following statements is true?</p> <p>(a) <math>3 &lt; \int_{D_1} f(x, y) d(x, y) &lt; \infty</math>.  (b) <math>2 &lt; \int_{D_1} f(x, y) d(x, y) \leq 3</math>.  (c) <math>1 &lt; \int_{D_1} f(x, y) d(x, y) \leq 2</math>.  (d) <math>0 &lt; \int_{D_1} f(x, y) d(x, y) \leq 1</math>.  (e) None of the above statements holds true.</p> <p>18. Let <math>D = \{(x, y) : x &gt; 0, y &gt; 0, 0 &lt; x^2 + y^2 \leq 1\}</math>. Which of the following statements is true?</p> <p>(a) <math>4 &lt; \int_D e^{-x^2-y^2} d(x, y) &lt; \infty</math>.  (b) <math>3 &lt; \int_D e^{-x^2-y^2} d(x, y) \leq 4</math>.  (c) <math>2 &lt; \int_D e^{-x^2-y^2} d(x, y) \leq 3</math>.  (d) <math>1 &lt; \int_D e^{-x^2-y^2} d(x, y) \leq 2</math>.  (e) None of the above statements holds true.</p> <p>19. Let <math>S_n = \sum_{k=1}^n \frac{k}{n^2} \sin\left(\frac{k}{n}\right)</math>. Which of the following statements is true?</p> <p>(a) <math>\lim_{n \rightarrow \infty} S_n/n</math> exists and <math>\lim_{n \rightarrow \infty} S_n/n \geq 1</math>.  (b) <math>\lim_{n \rightarrow \infty} S_n</math> exists and <math>\lim_{n \rightarrow \infty} S_n \leq \sqrt{2}</math>.  (c) <math>\lim_{n \rightarrow \infty} nS_n</math> exists and <math>\lim_{n \rightarrow \infty} nS_n \leq 1</math>.  (d) <math>\lim_{n \rightarrow \infty} n^2 S_n</math> exists and <math>\lim_{n \rightarrow \infty} n^2 S_n \leq 1/2</math>.  (e) None of the above statements holds true.</p> <p>20. Let <math>f(x, y) = x^2 y + \sin(2x + y)</math> for <math>x, y \in (-\infty, \infty)</math>. Which of the following statements is true?</p> <p>(a) <math>f_x(\pi, 0) \leq 1</math> and <math>f_{xx}(\pi, 0) \leq 5</math>.  (b) <math>f_x(\pi, 0) \leq 1</math> and <math>f_{xx}(\pi, 0) &gt; 5</math>.  (c) <math>f_x(\pi, 0) &gt; 1</math> and <math>f_{xx}(\pi, 0) \leq 5</math>.  (d) <math>f_x(\pi, 0) &gt; 1</math> and <math>f_{xx}(\pi, 0) &gt; 5</math>.  (e) None of the above statements holds true.</p>					
備 註	<p>一、作答於試題上者，不予計分。選擇題請在答案卡上作答，否則不予計分。  二、試題請隨卷繳交。</p>				