

1. (12 points) The reading scores of TOEIC (out of 495) in a school are distributed normally with a mean of 255 and a standard deviation of 106. The school awards a certificate of excellence to all students who score at least 350 on the reading exam of TOEIC. Suppose we pick one of the recognized students at random. What is the probability this student's score will be at least 400?
2. (14 points) Jack is considering leasing an inkjet printer. The total annual cost of the printer consists of a NT\$5,000 leasing fee, plus NT\$0.8 per page for ink and maintenance. The number of pages Jack prints in a year is a random variable with a mean 10,000 and standard deviation 2,000. Suppose that Jack leases the printer for three years and that the number of pages he prints each year is independent of previous years. Calculate the mean and variance of Jack's total costs over the three years.
3. (12 points) You are given the following hypotheses:
 $H_0 : \mu = 100$
 $H_A : \mu \neq 100$

We know that the sample standard deviation is 19 and the sample size is 12. For what sample mean would the p-value be equal to 0.05? Assume that all conditions necessary for inference are satisfied.

4. (12 points) A group of researchers wanted to understand the rate of growth for trees in a park. They took measurements of a random sample of 40 trees in 2010 and again measured those same trees in 2020. The data below summarize their measurements, where the heights are in meters:

	2010	2020	Differences
n	40	40	40
Mean	3.66	7.47	3.81
S.D.	1.07	2.90	2.19

Construct a 90% confidence interval for the average growth of trees in the park over 2010-2020.

5. (15 points) Assume that the true model is $y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + u_i$, but we only include x_i as the regressor to run a simple regression model. The OLS estimator can be written as

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})y_i}{\sum_{i=1}^n (x_i - \bar{x})^2}.$$

Assume that $E(u_i|x_i) = 0$, $\beta_2 < 0$, $Cov(x_i, z_i) = \sigma_{xz} > 0$, $Var(x_i) = \sigma_x^2$, and $Var(z_i) = \sigma_z^2$. Show that $\hat{\beta}_1$ is inconsistent, which is

$$\hat{\beta}_1 \xrightarrow{p} \beta_1 + Bias.$$

Please derive the bias term, *Bias*, and determine the sign of the bias term.

6. To estimate a demand function for fish, we use the data in the fish market and consider the following model:

$$\log(Q_t) = \beta_0 + \beta_1 \log(P_t) + \beta_2 MON_t + \beta_3 TUE_t + \beta_4 WED_t + \beta_5 THUR_t + u_t.$$

For the independent variables, we include a set of dummies for days of the week, and the based group is Friday. The following table shows the empirical result from ordinary least squares (OLS) regression:

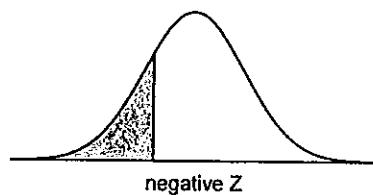
Variables	OLS $\log(Q_t)$
$\log(P_t)$	-0.525*** [0.176]
MON_t	-0.311 [0.226]
TUE_t	-0.683*** [0.223]
WED_t	-0.534** [0.220]
$THUR_t$	0.0672 [0.220]
Constant	8.244*** [0.163]
Observations	97
R-squared	0.217

Standard errors in brackets

*** p<0.01, ** p<0.05, * p<0.1

- (a) (5 points) Interpret the coefficient on $\log(P_t)$.
- (b) (5 points) Find the 95% confidence interval for β_1 . Then, test $H_0 : \beta_1 = 0$ against the two-sided alternative. What do you conclude?
- (c) (5 points) Why do we need to include the dummies for days of the week? Also, interpret the coefficients on those dummies.
- (d) (5 points) Explain why the variable $\log(P_t)$ could be an endogenous variable, and find a suitable instrumental variable (IV).
- (e) (5 points) Assume that Z_{1t} and Z_{2t} are two valid instrumental variables. Describe how you use two stage least squares (2SLS) to get the unbiased estimator. Write down the detailed procedures.
- (f) (5 points) Compared with the OLS estimator, do you think that the 2SLS estimator should be larger, smaller, or uncertain? Explain.
- (g) (5 points) If you want to check whether there exists a weak IV problem or not, what can you do? Write down the detailed procedures.

Table 1: Standard normal probability table

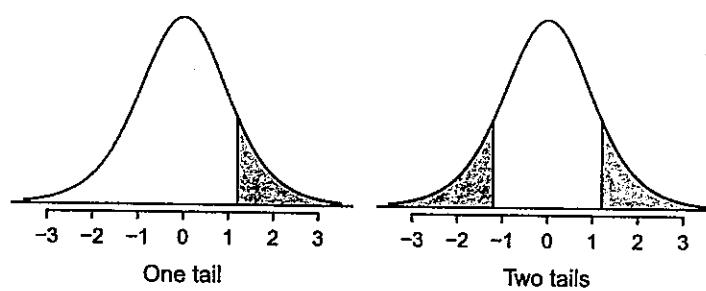


Second decimal place of Z										Z
0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	-3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	-3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	-3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	-3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	-3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	-2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	-2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	-2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	-2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	-2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	-2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	-2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	-2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	-2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	-2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	-1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	-1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	-1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	-1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	-1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	-0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	-0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	-0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	-0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	-0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	-0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	-0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	-0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	-0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	0.0

*For $Z \leq -3.50$, the probability is less than or equal to 0.0002.

見背面

Table 2: t distribution probability table ($df \leq 30$)



	one tail	0.100	0.050	0.025	0.010	0.005
	two tails	0.200	0.100	0.050	0.020	0.010
df	1	3.08	6.31	12.71	31.82	63.66
	2	1.89	2.92	4.30	6.96	9.92
	3	1.64	2.35	3.18	4.54	5.84
	4	1.53	2.13	2.78	3.75	4.60
	5	1.48	2.02	2.57	3.36	4.03
	6	1.44	1.94	2.45	3.14	3.71
	7	1.41	1.89	2.36	3.00	3.50
	8	1.40	1.86	2.31	2.90	3.36
	9	1.38	1.83	2.26	2.82	3.25
	10	1.37	1.81	2.23	2.76	3.17
	11	1.36	1.80	2.20	2.72	3.11
	12	1.36	1.78	2.18	2.68	3.05
	13	1.35	1.77	2.16	2.65	3.01
	14	1.35	1.76	2.14	2.62	2.98
	15	1.34	1.75	2.13	2.60	2.95
	16	1.34	1.75	2.12	2.58	2.92
	17	1.33	1.74	2.11	2.57	2.90
	18	1.33	1.73	2.10	2.55	2.88
	19	1.33	1.73	2.09	2.54	2.86
	20	1.33	1.72	2.09	2.53	2.85
	21	1.32	1.72	2.08	2.52	2.83
	22	1.32	1.72	2.07	2.51	2.82
	23	1.32	1.71	2.07	2.50	2.81
	24	1.32	1.71	2.06	2.49	2.80
	25	1.32	1.71	2.06	2.49	2.79
	26	1.31	1.71	2.06	2.48	2.78
	27	1.31	1.70	2.05	2.47	2.77
	28	1.31	1.70	2.05	2.47	2.76
	29	1.31	1.70	2.05	2.46	2.76
	30	1.31	1.70	2.04	2.46	2.75

題號：112

科目：統計學(A)

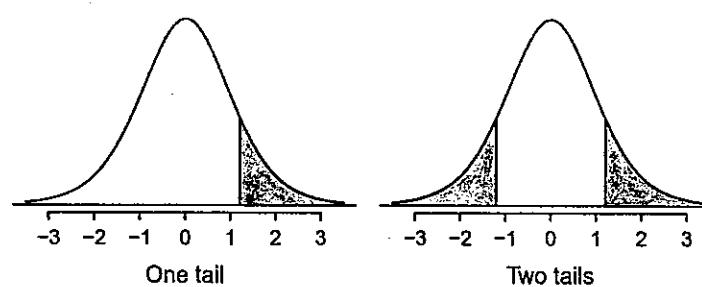
節次：4

國立臺灣大學 109 學年度碩士班招生考試試題

題號：112

共 7 頁之第 5 頁

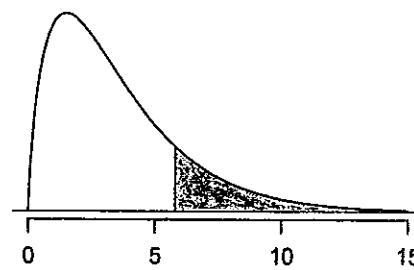
Table 3: t distribution probability table ($df \geq 31$)



	one tail	0.100	0.050	0.025	0.010	0.005
	two tails	0.200	0.100	0.050	0.020	0.010
df						
31		1.31	1.70	2.04	2.45	2.74
32		1.31	1.69	2.04	2.45	2.74
33		1.31	1.69	2.03	2.44	2.73
34		1.31	1.69	2.03	2.44	2.73
35		1.31	1.69	2.03	2.44	2.72
36		1.31	1.69	2.03	2.43	2.72
37		1.30	1.69	2.03	2.43	2.72
38		1.30	1.69	2.02	2.43	2.71
39		1.30	1.68	2.02	2.43	2.71
40		1.30	1.68	2.02	2.42	2.70
41		1.30	1.68	2.02	2.42	2.70
42		1.30	1.68	2.02	2.42	2.70
43		1.30	1.68	2.02	2.42	2.70
44		1.30	1.68	2.02	2.41	2.69
45		1.30	1.68	2.01	2.41	2.69
46		1.30	1.68	2.01	2.41	2.69
47		1.30	1.68	2.01	2.41	2.68
48		1.30	1.68	2.01	2.41	2.68
49		1.30	1.68	2.01	2.40	2.68
50		1.30	1.68	2.01	2.40	2.68
60		1.30	1.67	2.00	2.39	2.66
70		1.29	1.67	1.99	2.38	2.65
80		1.29	1.66	1.99	2.37	2.64
90		1.29	1.66	1.99	2.37	2.63
100		1.29	1.66	1.98	2.36	2.63
150		1.29	1.66	1.98	2.35	2.61
200		1.29	1.65	1.97	2.35	2.60
300		1.28	1.65	1.97	2.34	2.59
400		1.28	1.65	1.97	2.34	2.59
500		1.28	1.65	1.96	2.33	2.59
∞		1.28	1.65	1.96	2.33	2.58

見背面

Table 4: Chi-square probability table



df	Upper tail	0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83	
2	2.41	3.22	4.61	5.99	7.82	9.21	10.60	13.82	
3	3.66	4.64	6.25	7.81	9.84	11.34	12.84	16.27	
4	4.88	5.99	7.78	9.49	11.67	13.28	14.86	18.47	
5	6.06	7.29	9.24	11.07	13.39	15.09	16.75	20.52	
6	7.23	8.56	10.64	12.59	15.03	16.81	18.55	22.46	
7	8.38	9.80	12.02	14.07	16.62	18.48	20.28	24.32	
8	9.52	11.03	13.36	15.51	18.17	20.09	21.95	26.12	
9	10.66	12.24	14.68	16.92	19.68	21.67	23.59	27.88	
10	11.78	13.44	15.99	18.31	21.16	23.21	25.19	29.59	
11	12.90	14.63	17.28	19.68	22.62	24.72	26.76	31.26	
12	14.01	15.81	18.55	21.03	24.05	26.22	28.30	32.91	
13	15.12	16.98	19.81	22.36	25.47	27.69	29.82	34.53	
14	16.22	18.15	21.06	23.68	26.87	29.14	31.32	36.12	
15	17.32	19.31	22.31	25.00	28.26	30.58	32.80	37.70	
16	18.42	20.47	23.54	26.30	29.63	32.00	34.27	39.25	
17	19.51	21.61	24.77	27.59	31.00	33.41	35.72	40.79	
18	20.60	22.76	25.99	28.87	32.35	34.81	37.16	42.31	
19	21.69	23.90	27.20	30.14	33.69	36.19	38.58	43.82	
20	22.77	25.04	28.41	31.41	35.02	37.57	40.00	45.31	
25	28.17	30.68	34.38	37.65	41.57	44.31	46.93	52.62	
30	33.53	36.25	40.26	43.77	47.96	50.89	53.67	59.70	
40	44.16	47.27	51.81	55.76	60.44	63.69	66.77	73.40	
50	54.72	58.16	63.17	67.50	72.61	76.15	79.49	86.66	

題號：112
科目：統計學(A)
節次：4

國立臺灣大學 109 學年度碩士班招生考試試題

題號：112

共 7 頁之第 7 頁

Table 5: F distribution probability table for 5% significance level

F	Degrees of freedom in the numerator (df_1)										Degrees of freedom in the denominator (df_2)								
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.31
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.06	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.03	1.96	1.90	1.87	1.84	1.79	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Table 6: F distribution probability table for 10% significance level

F	Degrees of freedom in the numerator (df_1)										Degrees of freedom in the denominator (df_2)								
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19					