國立臺灣科技大學 108 學年度碩士班招生試題

系所組別：工業管理系碩士班甲組

科　　目：作業研究

(總分為 100 分)

1. **(20%)** Consider the following linear programming problem.

$$\begin{array}{lllllll} \text{Maximize} & Z = & 2x_1 & +4x_2 & +3x_3 & & \\ \text{Subject to} & 2x_1 & -x_2 & -3x_3 & \geq & -1 \\ & x_1 & +x_2 & +2x_3 & \leq & 4 \\ & x_1 \geq 0, & x_2 \geq 0, & x_3 \leq 0 & & \end{array}$$

(1) Construct and solve the dual problem of this primal problem. **(10%)**

(2) Use the optimal solution to the dual problem obtained in part (1) and the complementary slackness theorem to derive the optimal solution to the primal problem. **(10%)**

2. **(30%)** A company makes two products. Each unit of the first product requires 2 hours on machine 1 and 1 hour on machine 2. Each unit of the second product requires 1 hour on machine 1 and 3 hours on machine 2. Machine 1 is available only 10 hours per day and machine 2 only 7 hours per day. The profit per unit sold is 12 for the first product and 16 for the second. The amount of each product produced per day must be an integral multiple of 0.25. The objective is to determine the mix of production quantities that will maximize total profit.

(1) Formulate an integer programming model for this problem. **(10%)**

(2) Use the branch-and-bound algorithm to solve this problem. **(20%)**

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3. **(10%)** NTUST can plant either corn or soybeans. The probabilities that the next harvest prices will go up, stay the same, or go down are .25, .30, and .45, respectively. If the prices go up, the corn crop will net $30,000 and the soybeans will net $10,000. If the prices remain unchanged, NTUST will break even. If the prices go down, the corn and the soybeans crops will sustain losses of $35,000 and $5,000, respectively. Which crop should NTUST plant? **(10%)**

4. **(20%)** Tim must make five years worth of progress to complete his doctorate degree at NTUST. However, he enjoys the life of a student and is in no hurry to finish his degree. In any academic year, there is a 50% chance he may take the year off, and a 50% chance of pursuing the degree full time. After completing three academic years, there is a 30% chance that Tim may simply get a master's degree, a 20% chance of taking the next year off but continuing in the Ph.D. program, and 50% chance of attending school full time toward his doctorate.
   (1) Determine the probability that Tim will end his academic journey with only a master's degree. **(10%)**
   (2) If Tim's fellowship pays an annual stipend of $18,000 (but only when he attends school), how much will he be paid before ending up with a degree? **(10%)**

5. **(20%)** An operator attends 5 automatic machines. After each machine completes a batch run, the operator must reset it before a new batch is started. The time to complete a batch run is exponential with means 45 minutes. The setup time is also exponential with mean 8 minutes.
   (1) Determine the average number of machines that are awaiting setup or are being set up. **(10%)**
   (2) Compute the probability that all machines are working. **(5%)**
   (3) Determine the average time a machine is down. **(5%)**