

國立臺灣科技大學 108 學年度碩士班招生試題

系所組別：工業管理系碩士班甲組、乙組、丙組

科 目：統計學

(總分為 100 分)

(Total 100 points.) *There are 6 problems in this exam. Show intermediate steps and formulas for partial credit. You must explain how you compute your results or answers for full credit.*

1. (30 points)

- (a) Suppose that there are 10 blue balls and 30 green balls in an urn initially. One of the balls is drawn at random from the urn, and its color is recorded without putting back in the urn. Repeat this process 5 times. What is the probability that there are exactly 2 blue balls have been drawn? (5 points)
- (b) Suppose that there are 10 blue balls and 30 green balls in an urn initially. One of the balls is drawn at random from the urn, and its color is recorded. Then, it is put back in the urn. Repeat this process 5 times. What is the probability that there are exactly 2 blue balls have been drawn? (5 points)
- (c) Suppose that there are 10 blue balls and 30 green balls in an urn initially. One of the balls is drawn at random from the urn, and its color is recorded. But, when it is put back in the urn, 5 additional balls of the same color are put in with it. What is the probability that the second ball drawn is blue? (10 points)
- (d) From part (c), what is the probability that the first ball drawn was blue given that the second ball drawn was green? (10 points)

2. (10 points) The joint pdf of the two continuous random variables X and Y is given by $f(x, y) = e^{-y}$ for $0 < x < y < \infty$. The moment-generating function of the joint distribution is defined as $M(t_1, t_2) = E(e^{t_1 X + t_2 Y})$ if it exists. Using the moment-generating function, compute $E(X)$, $E(Y)$, and $\text{Cov}(X, Y)$.

3. (10 points) Suppose that the random variable $X = Y_1 + Y_2 + \dots + Y_N$ where Y_1, Y_2, \dots, Y_N are independent and identically distributed exponential random variables with mean μ , and N is geometric random variable with mean $1/p$ which is independent of Y_1, Y_2, \dots . What is the cumulative distribution function of X in terms of μ and p ?



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4. (15 points) A professor in Taiwan Tech wanted to compare the English proficiency of students from two different departments. 9 and 11 students from departments A and B, respectively, were randomly selected to determine whether the mean English proficiency scores varies from one department to another. The following table contains information regarding the scores of the students:

Department	Sample		Standard
	Size	Mean	Deviation
A	9	70	10
B	11	65	15

Assume that the scores are normally distributed.

- (a) Is there evidence of a difference in the variances of the score of students at the department A and at the department B? (Please clearly state the null hypothesis, the alternative hypothesis, and the test statistic, and use $\alpha = 0.10$) (5 points)
- (b) Please test the hypothesis that the mean English proficiency scores of students from different departments are equal. (Please clearly state the null hypothesis, the alternative hypothesis, and the test statistic, and use $\alpha = 0.05$) (10 points)
5. (20 points) Five students were asked to evaluate the service at each of the three restaurants in Taiwan Tech. The following data displays the scores rated by these students.

Student	Restaurant		
	A	B	C
1	70	80	60
2	65	70	60
3	55	60	50
4	60	60	60
5	50	55	45



Assuming that there is no interaction effect (between restaurant and student), Please

- (a) Construct the ANOVA table (10 points)
- (b) Determine whether there is evidence of a difference in the scores of the three restaurants (Please clearly state the null hypothesis, the alternative hypothesis, and the test statistic, and use $\alpha = 0.05$). (3 points)
- (c) Determine whether there is evidence of a difference in the scores rated by the five different students. (Please clearly state the null hypothesis, the alternative hypothesis, and the test statistic, and use $\alpha = 0.05$) (3 points)
- (d) Students in this study can be considered as "blocks". Please explain why randomized block design is better than the completely randomized design here? (4 points)

6. (15 points) For a random sample X_1, \dots, X_n of Bernoulli(p) variables, we want to test

$$H_0 : p = 0.49 \text{ versus } H_1 : p = 0.51$$

using a test function that rejects H_0 if $\sum_{i=1}^n X_i > c$. Please use the central limit theorem to determine, approximately, the sample size n needed and the critical value c so that both the Type I error and Type II errors are 0.01.

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Table A.3 Normal Probability Table

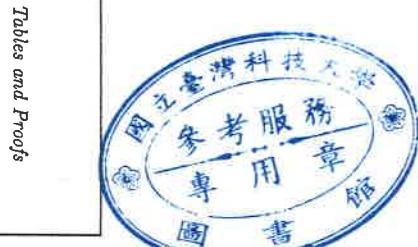
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Table A.3 Areas under the Normal Curve										
<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002
-3.3	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0020	0.0019	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0043	0.0041	0.0040	0.0039	0.0038	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0100	0.0099	0.0094	0.0091	0.0087	0.0084	0.0081
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0560
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0951	0.0934	0.0918	0.0901	0.0885	0.0853	0.0838	0.0823	0.0809	0.0793
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641	0.4601

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Table A.3 (continued) Areas under the Normal Curve

Appendix A Statistical Tables and Proofs



<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8315	0.8340	0.8365	0.8389	0.8419
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8543	0.8566	0.8586	0.8606	0.8629	0.8650	0.8672	0.8690	0.8710	0.8730
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9706	0.9714
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9931	0.9932	0.9934	0.9936	0.9938
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.99	

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Table A.4 Student t-Distribution Probability Table

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v	α					
	0.40	0.30	0.20	0.15	0.10	0.05
1	0.325	0.727	1.376	1.963	3.078	6.314
2	0.289	0.617	1.061	1.386	1.886	2.920
3	0.277	0.584	0.978	1.250	1.638	2.353
4	0.271	0.569	0.941	1.190	1.533	2.132
5	0.267	0.559	0.920	1.156	1.476	2.015
6	0.265	0.553	0.906	1.134	1.440	1.943
7	0.263	0.549	0.896	1.119	1.415	1.895
8	0.262	0.546	0.889	1.108	1.397	1.860
9	0.261	0.543	0.883	1.100	1.383	1.833
10	0.260	0.542	0.879	1.093	1.372	1.812
11	0.260	0.540	0.876	1.088	1.363	1.796
12	0.259	0.539	0.873	1.083	1.356	1.782
13	0.259	0.538	0.870	1.079	1.350	1.771
14	0.258	0.537	0.868	1.076	1.345	1.761
15	0.258	0.536	0.866	1.074	1.341	1.753
16	0.258	0.535	0.865	1.071	1.337	1.746
17	0.257	0.534	0.863	1.069	1.333	1.740
18	0.257	0.534	0.862	1.067	1.330	1.734
19	0.257	0.533	0.861	1.066	1.328	1.729
20	0.257	0.533	0.860	1.064	1.325	1.725
21	0.257	0.532	0.859	1.063	1.323	1.721
22	0.256	0.532	0.858	1.061	1.321	1.717
23	0.256	0.532	0.858	1.060	1.319	1.714
24	0.256	0.531	0.857	1.059	1.318	1.711
25	0.256	0.531	0.856	1.058	1.316	1.708
26	0.256	0.531	0.856	1.058	1.315	1.706
27	0.256	0.531	0.855	1.057	1.314	1.703
28	0.256	0.530	0.855	1.056	1.313	1.701
29	0.256	0.530	0.854	1.055	1.311	1.699
30	0.256	0.530	0.854	1.055	1.310	1.697
40	0.255	0.529	0.851	1.050	1.303	1.684
60	0.254	0.527	0.848	1.045	1.296	1.671
120	0.254	0.526	0.845	1.041	1.289	1.658
∞	0.253	0.524	0.842	1.036	1.282	1.645

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Table A.4 (continued) Critical Values of the t-Distribution

v	α					
	0.02	0.015	0.01	0.0075	0.005	0.0025
1	15.894	21.205	31.821	42.433	63.656	127.321
2	4.849	5.643	6.965	8.073	9.925	14.089
3	3.482	3.896	4.541	5.047	5.841	7.453
4	2.999	3.298	3.747	4.088	4.604	5.598
5	2.757	3.003	3.365	3.634	4.032	4.773
6	2.612	2.829	3.143	3.372	3.707	4.317
7	2.517	2.715	2.998	3.203	3.499	4.029
8	2.449	2.634	2.896	3.035	3.355	3.833
9	2.398	2.574	2.821	2.998	3.250	3.690
10	2.359	2.527	2.764	2.932	3.169	3.581
11	2.328	2.491	2.718	2.879	3.106	3.497
12	2.303	2.461	2.681	2.836	3.055	3.428
13	2.282	2.436	2.650	2.801	3.012	3.372
14	2.264	2.415	2.624	2.771	2.977	3.326
15	2.249	2.397	2.602	2.746	2.947	3.286
16	2.235	2.382	2.583	2.724	2.921	3.252
17	2.224	2.368	2.567	2.706	2.898	3.222
18	2.214	2.356	2.552	2.689	2.878	3.197
19	2.205	2.346	2.539	2.674	2.861	3.174
20	2.197	2.336	2.528	2.661	2.845	3.153
21	2.189	2.328	2.518	2.649	2.831	3.135
22	2.183	2.320	2.508	2.639	2.819	3.119
23	2.177	2.313	2.500	2.629	2.807	3.104
24	2.172	2.307	2.492	2.620	2.797	3.091
25	2.167	2.301	2.485	2.612	2.787	3.078
26	2.162	2.296	2.479	2.605	2.779	3.067
27	2.158	2.291	2.473	2.598	2.771	3.057
28	2.154	2.286	2.467	2.592	2.763	3.047
29	2.150	2.282	2.462	2.586	2.756	3.038
30	2.147	2.278	2.457	2.581	2.750	3.030
40	2.123	2.250	2.423	2.542	2.704	2.971
60	2.099	2.223	2.390	2.504	2.660	2.915
120	2.076	2.196	2.353	2.468	2.617	2.860
∞	2.054	2.170	2.326	2.432	2.576	2.807

Appendix A Statistical Tables and Proofs



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Table A.6 F-Distribution Probability Table

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Table A.6 Critical Values of the F-Distribution₁ $f_{0.05}(v_1, v_2)$