國立臺灣科技大學 108 學年度碩士班招生試題

系所組別:自動化及控制研究所碩士班

科 目:工程數學

(總分為100分)

1. Use the Fourier transform to solve

$$(15\%)$$

$$y'' + 6y' + 5y = \delta(t-3)$$

2. Solve

(15%)

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \text{ for } x > 0, \ t > 0$$

$$u(x, 0) = 0$$
,  $u(0, t) = t^2$ ,  $\lim_{x \to \infty} u(x, t) = 0$ 

3. Let

(20%)

$$p(z) = (z - z_1)(z - z_2) \cdots (z - z_n)$$

with  $z_1, \ldots, z_n$  distinct complex numbers. Let  $\gamma$  be a positively oriented closed path enclosing each  $z_j$ . Evaluate

$$\oint_{\gamma} \frac{p'(z)}{p(z)} dz$$

first by using the residue theorem and then by using the argument principle.

4. Use the matrix exponential to solve the initial value problem (15%)

$$Y' = AY, Y(0) = Y_0,$$

where 
$$A = \begin{bmatrix} 3 & 4 \\ 3 & 2 \end{bmatrix}$$
,  $Y_0 = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ .



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5. Find a power series solution for the differential equation (20%)

$$(x^2 + 1)y'' + xy' - y = 0.$$

6. Suppose the capacitor in the circuit of Figure P6 initially has a charge of zero and there is no initial current. At time t=2 second, the switch is thrown from position B to A, held there for 1 second, and then switched back to B. Please find the output voltage  $E_{\text{out}}$  on the capacitor. (15%)

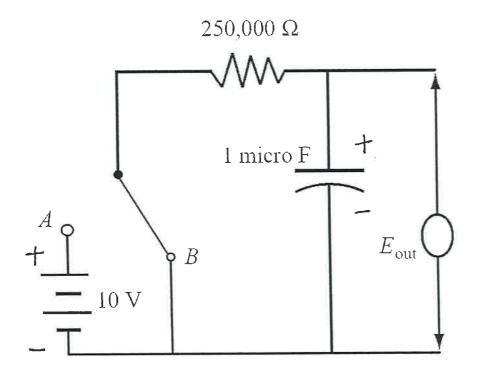


Figure P6