

國立臺灣師範大學 108 學年度碩士班招生考試試題

科目：機率與統計

適用系所：數學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。
3.答案必須有計算過程，否則會斟酌扣分。

1. (30 分) The foundations of probability are based upon three statements called axioms.

(a) State the three axioms of probability.

(b) Show that the probability of the empty set is zero.

(c) For any finite sequence of events $E_1, \dots, E_n$ that are mutually exclusive, i.e.

$$E_i \cap E_j = \emptyset \text{ if } i \neq j, \text{ show that } P\left(\bigcup_{i=1}^n E_i\right) = \sum_{i=1}^n P(E_i).$$

2. (30 分) The random vector (X, Y) has an absolutely continuous distribution with density

$$f(x, y) = \begin{cases} \frac{1}{\sqrt{\pi}} e^{-y} & \text{if } y > x^2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Determine the marginal density of X .

(b) Given x , determine $E(Y | X = x)$.

(c) Determine $E(Y)$.

3. (20 分) Let X have a Weibull distribution with density

$$f(x | \alpha, \beta) = \begin{cases} \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right] & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the mean and variance of X .

(b) Based on a random sample with $n = 20$, $\bar{x} = 28$, $\sum x_i^2 = 16505$, compute the method of moments estimates of α and β . (Hint: $\Gamma(1.2) = 0.918$, $\Gamma(1.4) = 0.887$, $[\Gamma(1.2)]^2 / \Gamma(1.4) = 0.95$, where $\Gamma(\cdot)$ is the gamma function.)

4. (20 分) Test the hypothesis $H_0: \lambda = 1$ against $H_a: \lambda = 0.1$ based on a random sample $Y_1, \dots, Y_{10}$ from the population of Poisson distribution with mean λ .

(a) Determine the most powerful α -test for $\alpha = 0.01$.

(b) Calculate for this test the probability of type II error.

(The cumulative Poisson distribution table is given on the next page)

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Table: Cumulative Distribution Function of Poisson Distribution, $P(X \leq x) = \sum_{w=0}^x e^{-\lambda} \frac{\lambda^w}{w!}$

CDF	$\lambda = E(X)$									
x	1	2	3	4	5	6	7	8	9	10
0	0.368	0.135	0.050	0.018	0.007	0.002	0.001	0.000	0.000	0.000
1	0.736	0.406	0.199	0.092	0.040	0.017	0.007	0.003	0.001	0.000
2	0.920	0.677	0.423	0.238	0.125	0.062	0.030	0.014	0.006	0.003
3	0.981	0.857	0.647	0.433	0.265	0.151	0.082	0.042	0.021	0.010
4	0.996	0.947	0.815	0.629	0.440	0.285	0.173	0.100	0.055	0.029
5	0.999	0.983	0.916	0.785	0.616	0.446	0.301	0.191	0.116	0.067
6	1.000	0.995	0.966	0.889	0.762	0.606	0.450	0.313	0.207	0.130
7	1.000	0.999	0.988	0.949	0.867	0.744	0.599	0.453	0.324	0.220
8	1.000	1.000	0.996	0.979	0.932	0.847	0.729	0.593	0.456	0.333
9	1.000	1.000	0.999	0.992	0.968	0.916	0.830	0.717	0.587	0.458
10	1.000	1.000	1.000	0.997	0.986	0.957	0.901	0.816	0.706	0.583
11	1.000	1.000	1.000	0.999	0.995	0.980	0.947	0.888	0.803	0.697
12	1.000	1.000	1.000	1.000	0.998	0.991	0.973	0.936	0.876	0.792
13	1.000	1.000	1.000	1.000	0.999	0.996	0.987	0.966	0.926	0.864
14	1.000	1.000	1.000	1.000	1.000	0.999	0.994	0.983	0.959	0.917
15	1.000	1.000	1.000	1.000	1.000	0.999	0.998	0.992	0.978	0.951
16	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.996	0.989	0.973
17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.995	0.986
18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.998	0.993
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.997
20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998
21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999
22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000