國立臺北大學 108 學年度碩士班一般入學考試試題

系（所）組別：金融與合作經營學系

科　　目：統計學

☐可 ☑不可使用計算機

一、A finance PhD program recruits new students by two ways: Choosing the good-grade postgraduates (Inner Promotion, IP), or by opening exam competition (Open Competition, OC). $P(IP) = 0.4$, $P(OC) = 0.6$. After the first year qualification test, there are 90% IP and 75% OC students passed. If you randomly select a test-passed student, what is the probability the student recruited from IP? (10 %)

二、A mayor wants to know whether there exists seasonal differences about the visitors travelling to his city. He selects 1360 passengers and surveys their arrival time:

Season	Spring	Summer	Autumn	Winter
Numbers	334	371	327	328

Please use Chi-square test under the significant level α=0.01. (13%)

$\chi^2_\nu(\alpha)$					
	α				
ν	.10	.05	.025	.01	.005
1	2.706	3.843	5.025	6.637	7.882
2	4.605	5.992	7.378	9.210	10.597
3	6.251	7.815	9.348	11.344	12.837
4	7.779	9.488	11.143	13.277	14.860
5	9.236	11.070	12.832	15.085	16.748

三、You participate a game by throwing a coin, which will be HEAD with P(H)=0.6 and TAIL with P(T)=0.4. The throwing results are independent. In the beginning, you own 5 dollars. If you throw a HEAD, you can get 1 dollar; however, if you throw a TAIL, you lose 1 dollar. This game will end when you own 15 dollars or lose all the money. What is the probability you own 15 dollars and lose all the money respectively? (12 %)

四、Two independent random variables X and Y. Y follows standard normal distribution, and X is uniform between [0,1]. Z = X + Y. Please derive the detail process that $cov(X, Z) = var(X)$. (15%)

五、Suppose $X_1, X_2, \ldots, X_n$ are independent random variables and follow Poisson distribution with mean λ. Please use moment generating function to show that $\bar{X} = \sum X_i / n$ converges in probability to λ. (Hint: The distribution of $\bar{X}$ degenerates to a constant random variable.) (10%)

六、Suppose $X_1, X_2, \ldots, X_{10}$ are independent random variables and follow $N(0, 1)$. Let $S = X_1+X_2+X_3+X_4+X_5$ and $W = X_6^2+X_7^2+X_8^2+X_9^2+X_{10}^2$. What is the probability $P(S/\sqrt{W} > 4.0324)$? (Rounded to third place.) (10%)

七、An urn contains six balls, one marked WIN and five marked LOSE. You and another player take turns selecting a ball from the urn, one at a time. The first person to select the WIN ball is the winner. If you draw first, find the probability that you will win

(a) When the sampling is done with replacement. (10%)

(b) When the sampling is done without replacement. (5%)

八、Bowl B_1 contains 2 white chips, bowl B_2 contains 2 red chips, bowl B_3 contains 2 white and 2 red chips, and bowl B_4 contains 3 white chips and 1 red chip. The probabilities of selecting bowl B_1, B_2, B_3, or B_4 are 1/2, 1/4, 1/8, and 1/8, respectively. A bowl is selected using these probabilities, and a chip is then drawn random. Find

(a) P(W), the probability of drawing a white chip. (10%)

(b) $P(B_1|W)$, the conditional probability that bowl B_1 had been selected, given that a white chip was drawn. (5%)

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附表：

$$P(F \le f) = \int_0^f \frac{\Gamma[(r_1 + r_2)/2](r_1/r_2)^{r_1/2} w^{r_1/2-1}}{\Gamma(r_1/2)\Gamma(r_2/2)(1 + r_1 w/r_2)^{(r_1+r_2)/2}}\, dw$$

α	$P(F \le f)$	Den. d.f. r_2	Numerator Degrees of Freedom, r_1							
			1	2	3	4	5	6	7	8
0.05	0.95	1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9
0.025	0.975		647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66
0.01	0.99		4052	4999.5	5403	5625	5764	5859	5928	5981
0.05	0.95	2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37
0.025	0.975		38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37
0.01	0.99		98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37
0.05	0.95	3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85
0.025	0.975		17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54
0.01	0.99		34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49
0.05	0.95	4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04
0.025	0.975		12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98
0.01	0.99		21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80
0.05	0.95	5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82
0.025	0.975		10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76
0.01	0.99		16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29
0.05	0.95	6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15
0.025	0.975		8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60
0.01	0.99		13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10

試題隨卷繳交