※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Part A: Multiple-Choice Questions (50 points, 10 points each)

1. Let $f(x) = 5x^2 + 6x - 1$. Suppose that $y = mx + b$ is the tangent line passing through point $(-1, -2)$. Then
   (a) $m = 6,\ b = 8$.
   (b) $m = -6,\ b = -8$.
   (c) $m = -4,\ b = -6$.
   (d) $m = 4,\ b = 2$.

2. Let $f(x) = x^4 + x^3 - 3x^2 + 1$. Then
   (a) $f(x)$ is increasing on $(-3,\ 0)$;
   (b) $f(0) = 1$ is a relative minimum;
   (c) the graph of $f(x)$ is concave upward on $(\frac{1}{2},\ 2)$;
   (d) none of the above.

3. Consider a cubic polynomial
   $$p(x) = a \cdot \frac{x(1-x)}{2} + b \cdot (x+1)(1-x) + c \cdot \frac{x(x+1)}{2} + t \cdot x(x+1)(1-x).$$
   Suppose that the graph of $p(x)$ passes through points $(-1,5), (0,9)$ and $(1,7)$, where coefficients a, b, c and t are constant. Then
   (a) $a = 5$;
   (b) $b = 9$;
   (c) $c = 7$;
   (d) $\int_{-1}^{1} p(x)dx = 10$.

4. Let $F(x,y) = \ln(x^2 + xy + y^2)$. Then
   (a) $F_x(-1,4) = \frac{2}{13}$, where F_x denotes the partial derivative of $F(x,y)$ with respect to the variable x,
   (b) $F_y(-1,4) = \frac{2}{13}$, where F_y denotes the partial derivative of $F(x,y)$ with respect to the variable y,
   (c) $\mathbb{R}^2 = \{(x,y) | x, y \in \mathbb{R}\}$ gives the domain of $F(x,y)$,
   (d) all of the above are correct.

5. In the following, which statement is true?

   (a) $\sum_{n=1}^{\infty}\frac{2n}{\sqrt{4n^2+3n+1}}$ is convergent,

   (b) $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\cdots$ is a convergent series,

   (c) $\sum_{n=0}^{\infty}\frac{7n}{3^n}$ is convergent.

   (d) $\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}x^{n+1}$ gives a Taylor's expansion series of $\ln(1+x)$ for $x\in(0,2)$.

Part B: Please simplify your answers as possible as you can. (50 points)

6. Consider a logistic distribution function $F(x)=\frac{1}{1+e^{-(x-\mu)/\gamma}}$, where $\mu\in\mathbb{R}$, and $\gamma>0$ are both constant. Show that

   (a) [2 points] $f(x)=F'(x)=\frac{e^{-(x-\mu)/\gamma}}{\gamma(1+e^{-(x-\mu)/\gamma})^2}$,

   (b) [3 points] $\int_{-\infty}^{\infty}f(x)dx=1$,

   (c) [5 points] the graph of $f(x)$ is symmetric about the axis $x=\mu$,

   (d) [5 points] the graph of $F(x)$ is symmetric about the point $\left(\mu,\frac{1}{2}\right)$.

7. [7 points] A company expects its continuous income function $c(t)$ during the next 4 years to be modeled by $c(t)=10000e^{0.05t}$ which means that the average rate of the annual interest is about 5%. Assume that the average inflation rate is of 4%, what is the present value of this income?

8. Consider $f(x)=\ln((x+1)^2\sqrt{2x+3})$ for $x>0$.
   (a) [5 points] Compute $f'(x)$,

   (b) [5 points] evaluate $\int_0^1 f(x)dx$.

9. [8 points] Assuming that x,y and z are positive, find the maximum of

$$f(x,y,z) = 3x + 2y + z$$

subject to the constraint

$$2x^2+y^2+z^2=6$$

10. [10 points] A corporation invests part of its revenue at a rate of P dollars per year in a fund for future corporate expansion. The fund earns an annual interest rate of r (in decimal form) compounded continuously. The rate of growth of the amount A (in dollars) in the fund is

$$\frac{dA}{dt} = rA + P,$$

where t is the time (in years). Solve the differential equation for $A = A(t)$ as a function of of t where $A = 0$ when $t = 0$.

Reference

Ron Larson and Tzuwei Cheng (2014), Calculus: An Application Approach
Bill Armstrong and Don Davis (2014), Brief Calculus for the Business, Social, and Life Sciences