考試科目	統計學B	系所別	金融學系財務工程與金融科技組	考試時間	2月18日(一)第2節

1. (25 points)

   (a) Let X be a real-valued random variable with density function f given by:

$$f(x) = \begin{cases} k(9-x^2) & \text{if} \quad 0 \le x \le 3 \\ 0 & \text{else} \end{cases} \tag{1}$$

   for some constant $k > 0$.

   1. (5 points) Find the value of k such that f is indeed a density function (i.e. it must integrate to 1).
   2. (5 points) Evaluate $E(X)$.
   3. (5 points) Evaluate $Var(X)$.

   (b) Let Y be a real-valued random variable with $E(Y) = \frac{5}{4}$ and $Var(Y) = \frac{101}{80}$.

   1. (5 points) Evaluate $E(XY)$.
   2. (5 points) Evaluate $Var(X+Y)$.

2. (25 points) Suppose that X and Y are two independent random variables which are exponentially distributed with parameter $\lambda = 1$. That is, X and Y both have Exponential(1)-distribution, with corresponding density functions being f_X and f_Y defined as $f_X(x) = e^{-x} \cdot 1_{\{x \ge 0\}}$ and $f_Y(y) = e^{-y} \cdot 1_{\{y \ge 0\}}$ respectively.

   Let $T = X + Y$.

   (a) (5 points) Find the joint density of the random vector (X, Y).

   (b) (10 points) Find the joint density of the random vector (X, T).

   (c) (5 points) Find the marginal density of T.

   (d) (5 points) Find the conditional density of X given $T = 1$.

3. (25 points) Let X be a normally distributed random variable with an unknown mean μ and a known variance equal to 1. Suppose that a prior distribution for μ is available, which is standard-normal. Assume further that we observe $X = x$ in an experiment. Derive the posterior distribution of μ given $X = x$ and hence determine the Maximum a Posteriori estimate of μ given $X = x$.

4. (25 points) Recall that for a Pareto distribution with parameters γ and m, its density function can be defined as follows:

$$f(x) = \begin{cases} \frac{\gamma m^\gamma}{x^{\gamma+1}} & \text{if } x \ge m \\ 0 & \text{else} \end{cases} \tag{2}$$

   Consider, in modeling insurance claims, one observes the arrivals of n claims with values $x_1, x_2, ..., x_n$. For $i = 1, ..., n$, assume that each x_i follows a Pareto distribution.

   (a) (10 points) Define and compute the likelihood function, $L(\gamma, m | x_1, x_2, ..., x_n)$, and hence compute the log-likelihood function.

   (b) (10 points) For a fix m, derive the maximum likelihood estimate for the parameter γ.

   (c) (5 points) For a fixed γ, find the maximum likelihood estimate for the parameter m.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。