考試科目	統計學 A	系所別	金融學系金融管理組	考試時間	2 月 18 日(一) 第 3 節

Part 1: Short Answer Questions

Mark the blank number and write your answer on the answer sheet. Do NOT provide any details. Each blank worths 5 points.

- If X and Y are independent standard normal distributions, the distribution of $U = X/Y$ is ___(1)___, and its moment generating function is ___(2)___.

- If the random variable X is $N(\theta, \theta)$, it is clear that $E(X^2) =$ ___(3)___, and $(1/n)\sum_{i=1}^{n} X_i^2$ converges in probability to ___(4)___.

- Consider a random sample of size n from the distribution $f(x) = \exp(-x)$, $0 < x < \infty$. If $\overline{X}$ is the mean of this random sample and $Y = \sqrt{n}\left(\overline{X} - 1\right)$, the asymptotic distribution of Y is ___(5)___. If $Z = \sqrt{n}\left(\sqrt{\overline{X}} - 1\right)$, the asymptotic distribution of Z is ___(6)___.

- Consider a random sample of size n from the distribution $f(x) = \theta(1-x)^{\theta-1}$, $0 < x < 1$, $\theta > 0$. The maximum likelihood estimator for θ is ___(7)___, and the likelihood ratio statistic for testing $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$ is ___(8)___.

- Consider a liner model $Y_i = \beta_0 + u_i$, $i = 1, 2, ..., n$ and $u_i \sim N(0, \sigma_u^2)$ is independent over i. The maximum likelihood estimator for σ_u^2 is $\widehat{\sigma}_u^2 =$ ___(9)___, and $Var\left(\widehat{\sigma}_u^2\right) =$ ___(10)___.

- Consider a model $\Pr(Y_i = 1|X_{1i}, X_{2i,...,}X_{ki}) = \Lambda(\beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + ... + \beta_k X_{ki})$ where Y is a binary variable and
$$\Lambda(z) = \frac{\exp(z)}{1 + \exp(z)}.$$
The log-likelihood of this model is -401.25 while that of the model with intercept only is -498.65. The average marginal effect of X_1 on Y can be expressed by ___(11)___, and the pseudo R^2 is ___(12)___.

- The return of a financial asset can be described by $r_t = 0.002 + \epsilon_t + 0.05\epsilon_{t-1}$ where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ is a random sequence. The theoretical correlation of r_t and r_{t-1} is ___(13)___, and the long-run sharpe ratio for this asset is ___(14)___.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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Part 2: Long Answer Questions

Be sure to provide all necessary details with your answers. Each question worths 10 points.

1. Consider a random sample Y_i of size n with the unknown population mean μ_Y. Explain how to test the hypothesis $\mu_Y = c$ by running a linear regression and show that this test statistic is exactly the same as the usual $\sqrt{n}\left(\overline{Y} - c\right)/s_Y$, where $\overline{Y}$ is the sample mean and s_Y is the sample standard deviation.

2. Let M_{t+1} be the stochastic discount factor and C_t be the aggregate consumption. The process for consumption growth is

$$\ln\left(\frac{C_{t+1}}{C_t}\right) = \mu_c + \sigma_c \epsilon_{c,t+1},$$

where $\epsilon_{c,t+1} \sim N(0,1)$. A classical asset pricing theory states that

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma},$$

and

$$R^f_{t+1} = \frac{1}{E_t\left(M_{t+1}\right)} - 1,$$

where R^f_{t+1} is the risk-free rate. If $\beta = 0.99$, $\gamma = 5$, $\mu_c = 0.0016$ and $\sigma_c = 0.0025$, what is the risk-free rate implied by this theory?

3. The table below displays the performance of a model in predicting economic recessions.

Cut point	Sensitivity	Specificity
0.2	0.9718	0.1517
0.3	0.8767	0.3608
0.4	0.6527	0.5898

Calculate the area under the Receiver Operating Characteristic Curve and comment on the performance of this model.

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