●不可使用電子計算機

1. (25%) Find the differential equation of the family of circles passing through (0, -3) and (0, 3), whose centers are on the x-axis.

2. (25%) Solve y'''-2y''+y' = xe^x + 5, y(0) = 2, y'(0) = 2, y''(0) = -1.

3. (25%) Use Laplace Transform to solve the following equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad x>0, \ t>0,$$

$$u(0,t)=1, \ \lim_{x\to\infty}u(x,t)=0, \ t>0, \ u(x,0)=e^{-x}, \ \left.\frac{\partial u}{\partial t}\right|_{t=0}=0, \ x>0$$

4. (25%) The temperature at a point (x, y, z) in space is inversely proportional to the square of the distance from (x, y, z) to the origin. It is known that $T(0, 0, 1) = 500$. Find the rate of change of T at (2, 3, 3) in the direction of (3, 1, 1). In which direction from (2, 3, 3) does the temperature increase most rapidly? At (2, 3, 3) what is the maximum rate of change of T?