

長庚大學107學年度研究所碩士班招生考試試題

系所：工商管理學系碩士班

考試科目：微積分

注意：請詳細閱讀下列試題，並請標明題號依試題順序將答案書寫於答案卷上。

本試題共 / 頁：第 / 頁

計算題 (總分 100 分，每題 10 分，共 10 題)

必須寫出計算過程，否則不予計分

- Find $\lim_{n \rightarrow \infty} \left(2^n \sqrt{2 - 2 \cos \left(\frac{\pi}{3 \cdot 2^n} \right)} \right)$ if it exists or otherwise indicate "does not exist".
- Find a power series representation of the function $f(x) = \frac{1}{1-x}$ and determine its interval of convergence.
- Determine whether the improper integral $\int_1^{\infty} \frac{\sin x}{x} dx$ converges or not.
- Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}$.
- Find $\frac{\partial w}{\partial v}$ when $u = 1, v = 1$, if $w = \sqrt{x^2 + y^2 + z^2}, x = u \sin v, y = u \cos v, z = uv$.
- Use total differential to approximate the quantity $\ln[(2.95)^2 - (2.05)^3]$.
- Find the equation of the tangent plane to the surface $xyz^3 + yz^2 = 4$ at the point $(1, 2, 1)$.
- Find and classify the critical points of the function $f(x, y) = 4xy - x^4 - y^4$ as yielding relative maxima, relative minima, saddle points, or none of these.
- Find the minimum distance from the origin to the surface $(x - y)^2 - z^2 = 1$.
- Evaluate the double integral $\iint_{\Omega} (x^2 + y^2) dx dy$ where $\Omega = \left\{ (x, y) \mid \frac{x^2}{9} + \frac{y^2}{16} \leq 1 \right\}$.