長庚大學107學年度研究所碩士班招生考試試題

系所：電機工程學系碩士班　　考試科目：線性代數

注意：請詳細閱讀下列試題，並請標明題號依試題順序將答案書寫於答案卷上。　本試題共 1 頁：第 1 頁

線性代數: 每大題二十分，各小題分數標於題後。請提供完整解題過程，否則不計分。

1. Let the matrix $A=\begin{bmatrix}1&2&3\\2&5&3\\1&0&8\end{bmatrix}$, and let C_{ij} and M_{ij} denote the cofactor and minor of the entry a_{ij} of the matrix A, respectively. Then,

   (a) A^{-1}=? (6%)

   (b) $3C_{11}+3C_{12}+8C_{13}$=? (7%)

   (c) $1(-1)^{2+1}M_{11}+2(-1)^{2+2}M_{12}+3(-1)^{2+3}M_{13}$=? (7%)

2. Let the matrices $E_1=\begin{bmatrix}1&0&0\\0&1&0\\-2&0&1\end{bmatrix}$, $E_2=\begin{bmatrix}1&0&0\\0&1&0\\0&0&\frac{1}{2}\end{bmatrix}$ and $E_3=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$. Then

   (a) $(E_3E_2E_1)^{-1}$=? (10%)

   (b) adj $(E_3E_2E_1)$=? (10%)

3. Let $A=\begin{bmatrix}1&2&-1&1\\2&4&-3&0\\1&2&1&5\end{bmatrix}$.

   (a) Find a basis for the row space of A. (5%)

   (b) Find a basis for the column space of A. (5%)

   (c) Find a basis for the null space of A. (10%)

4. Let $S=\{1,1+x,1+x+x^2\}$ and $T=\{1,2x,3x^2\}$ be two bases for P_2, where the vector space P_2 represents the set of all polynomials of degree 2 or less, and P_2 is a vector space with usual polynomial addition and scalar multiplication. Let $p(x)=3+2x+x^2$, then

   (a) $[p(x)]_S$=? (7%)

   (b) The transition matrix from S to T=? (7%)

   (c) $[p(x)]_T$=? (6%)

5. Let the vectors $\vec{u}=(1,2,-1,0)$, $\vec{v}=(2,5,-3,2)$, $\vec{w}=(2,4,-2,0)$ and $\vec{x}=(3,8,-5,4)$ be four vectors in the four-dimensional real space $\mathbf{R}^4$.

   (a) Find a basis for $span(\{\vec{u},\vec{v},\vec{w},\vec{x}\})$. (15%)

   (b) $\dim(span(\{\vec{u},\vec{v},\vec{w},\vec{x}\}))$=? (5%)