

元智大學 107 學年度 碩士班 招生試題卷

系(所)別：工業工程與管理 組別：不分組
學系碩士班

科目：機率與統計

用紙第 1 頁共 2 頁

●不可使用電子計算機

Final answers without meaningful intermediate steps will receive NO credits.

1. Let W be a random variable giving the number of heads minus the number of tails in four tosses of a fair coin.
 - (a) (8 points) Find the probability distribution function of the random variable W .
 - (b) (3 points) Find $E(3W + 5)$.
 - (c) (4 points) Find $\text{Var}(3W + 5)$.
2. In a certain industrial facility, accidents occur infrequently. It is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other. Let X be the number of accidents occurred on any given period of 600 days.
 - (a) (5 points) Name the distribution of X , and list the parameter values of the distribution.
 - (b) (5 points) Which distribution can be used as an approximate distribution of X ? Give the probability distribution function of this approximate distribution.
 - (c) (5 points) Based on the approximate distribution in (b), find the probability that there are at most three days with an accident each day.
3. Flip a fair coin sequentially and independently.
 - (a) (5 points) Find the expected number of flips until the first head appears.
 - (b) (5 points) Find the expected number of flips until the second head appears.
 - (c) (10 points) Find the expected number of flips until two heads appear consecutively.
4. (10 points) Consider a random sample X_1, X_2, \dots, X_n from the pdf

$$f(x; \theta) = \frac{1 + \theta x}{2}, \quad -1 \leq x \leq 1$$

where $-1 \leq \theta \leq 1$. Show that $3\bar{X}$ is an unbiased estimator of θ , where $\bar{X} = (\sum_{i=1}^n X_i)/n$.

5. The goal is to guess the gender (male or female) by observing the height of a person selected randomly from Yuan Ze University. Test at 10% significance level H_0 : the person is a male against H_1 : the person is a female. Let H denote the height of the selected person.
 - (a) (3 points) Denoting by k the critical value, which of the following is an appropriate critical region of the test?
 $\{(H < k) \text{ or } \{H > k\}\}$ Support your answer.
 - (b) (5 points) What do we need to know to determine the critical value k ? How to determine the critical value k ?
 - (c) (3 points) Suppose $H = 165$ cm, give the probability expression of p -value.
 - (d) (4 points) If the answer is in favor of H_0 , which type error will occur potentially? Express the probability of making such type error.

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- (e) (5 points) If we test at 5% significance level instead of 10%, how does the value of k change, larger, smaller or same? Explain your reasoning. How does the probability in question (d) change, larger, smaller, or same? Explain your reasoning.
6. Suppose that a random sample of 50 male students of Yuan Ze University is selected and the height of each student is determined. Let μ denote the average height of all male students in Yuan Ze University. Suppose that the resulting 95% confidence interval is (168.5 cm, 171.5 cm).
- (a) (5 points) Would a 90% confidence interval calculated from this same sample have been narrower or wider than the given interval? Explain your reasoning.
- (b) (5 points) Consider the following statement: We can be highly confident that 95% of all male students in Yuan Ze University have a height that is between 168.5 cm and 171.5 cm. Is this statement correct? Why or why not?
- (c) (5 points) Consider the following statement: There is a 95% chance that μ is between 168.5 cm and 171.5 cm. Is this statement correct? Why or why not?
- (d) (5 points) Consider the following statement: If the process of selecting a sample of size 50 and then computing the corresponding 95% interval is repeated 100 times, 95 of the resulting intervals will include μ . Is this statement correct? Why or why not?