

元智大學 107 學年度 碩士班 招生試題卷

系(所)別：電機工程學系碩士班 組別：甲組

科目：工程數學

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●不可使用電子計算機

1. (a) If A is a nonsingular matrix, prove that $A^T A$ is positive definite (10%)
 (b) If u , v , and w are linear independent vectors, show that u , $u+v$, and $u+v+w$ are also linear independent vectors. (10%)

2. Find the eigenvalues and eigenvectors of the matrix (10%)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

3. A and B are two independent events. $P(A)=0.4$, $P(B)=0.7$.

- (a) Find $P(A \cap B)$. (5%)
 (b) Find $P(A \cup B)$. (5%)

4. Let X be a number uniformly distributed in $[-1,1]$, $Y=X+X$:

- (a) Find $P(|X|<0.5)$ (5%)
 (b) Find $P(|Y|<1)$ (5%)

5. Given the following ODE, solve y . (12%)

$$y'' - y' + 2y = 2 \cos 2x - 4 \sin 2x, \quad y(0) = 0, \quad y'(0) = 0$$

6. According to Fig.1, given the periodic continuous-time waveform $f(t)$. (13%)

$$f(t) = \begin{cases} 0 & \text{for } -\pi < t < 0 \\ \sin t & \text{for } 0 < t < \pi \end{cases}, \text{ find its Fourier Series.}$$

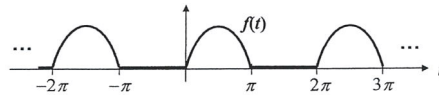


Fig.1

7. Solve the following ODE by Laplace transform. (12%)

$$y'' + y' = e^{t-2\pi} \cos(t-2\pi) u(t-2\pi)$$

8. Given the following systems of ODE, solve x_1 , x_2 and x_3 . (13%)

$$\begin{aligned} x_1' &= 2x_1 - 2x_2 + 3x_3, & x_1(0) &= 1 \\ x_2' &= x_1 + x_2 + x_3, & x_2(0) &= 0 \\ x_3' &= x_1 + 3x_2 - x_3, & x_3(0) &= 1/2 \end{aligned}$$