微積分試題共有 10 題，每題 10 分，共計 100 分。請寫出計算過程，否則不予計分。
在答案紙上請清楚標示題號。

Q1: Please find the indefinite integral: (10 points)

$$\int \frac{e^{-x}}{1+e^{-x}}\,dx$$

Q2: Please evaluate the integral over the region: (10 points)

$$\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1}\,dxdy$$

Q3: Determine the convergence or divergence of the given series:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^5}$: (5 points)

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n2^n}{n!}$: (5 points)

Q4: Evaluate the integral: (10 points)

$$\int \sin^2 4x \cos 4x\,dx$$

Q5: Find the general solution to the first-order linear differential equation: (10 points)

$y'+2y=\sin x$

Q6: Given $\dot{y}=dy/dt$, please derive the general solution of the following equation: (10 points)

$\dot{y}=y(a-by)$

Q7: Please find the general solution of the equation: (10 points)

$\ddot{y}-2\dot{y}-3y=9t^2$

Q8: Consider the following Cobb-Douglas production function $Q=4K^{3/4}L^{1/4}$.

(a) Please compute all third order partial derivatives of the production function Q (5 points)

(b) Suppose that the inputs K and L vary with time t and the interest rate r, via the expressions

$K(t,r)=\dfrac{10t^2}{r}$, and $L(t,r)=6t^2+250r$, Please calculate the rate of change of output Q with respect to t when t=10 and r=0.1.　(5 points)

Q9: In what direction should one move from the point $(2,3)$ to increase $4x^2y$ most rapidly? Present your answer as a vector of length 1. (10 points)

Q10: For the following functions defined on $\mathbf{R}^3$, find the critical points and classify them as local max, local min, saddle point, or "can't tell". (10 points)

$x^2+6xy+y^2-3yz+4z^2-10x-5y-21z$

試題隨卷繳回