國立臺北大學107學年度碩士班一般入學考試試題

系（所）組別：統計學系

科　　　　目：數理統計

☐可　☑不可使用計算機

1. (40%) Let $X_1, X_2, \cdots, X_n$ be a random sample from the distribution with a probability density function

$$f(x;\theta) = \theta(1-x)^{\theta-1}, 0 < x < 1;\ \ \theta > 0.$$

(a) (8%) Find the maximum likelihood estimator $T_n(X)$of θ, where $X = (X_1, X_2, \cdots, X_n)'$.

(b) (8%) Find the maximum likelihood estimator of the median.

(c) (8%) Find the Rao-Cramèr lower bound for the variance of $T_n(X)$.

(d) (8%) Based on $T_n(X)$, find the unique minimum variance unbiased estimator of θ.

(e) (8%) Consider $H_0: \theta = 2$ versus $H_a: \theta > 2$. Suppose a random sample of size 50 is collected as follows:

0.68 0.21 0.53 0.06 0.47 0.58 0.56 0.29 0.81 0.42 0.85 0.09 0.97

0.57 0.17 0.64 0.49 0.41 0.59 0.47 0.18 0.17 0.31 0.41 0.58 0.66

0.74 0.49 0.71 0.50 0.45 0.23 0.68 0.60 0.06 0.91 0.63 0.57 0.49

0.34 0.59 0.46 0.29 0.34 0.15 0.78 0.37 0.44 0.85 0.38

We find $50 - \sum_{i=1}^{50} x_i = 25.77$ and $\sum_{i=1}^{50} \log(1-x_i) = -17.43$. Construct a Wald-type test to these hypotheses. Use this test to make inference about this sample given the significance level 0.05.

2. (10%) Let $X_1, X_2, \cdots, X_n$ be a random sample from the distribution with a probability density function

$$f(x;\theta) = \theta_1 e^{-\theta_1(x-\theta_2)}, x > 0;\ \ \theta_1 > 0; \theta_1 \in R.$$

Find the maximum likelihood estimators of $\theta_1$and θ_2.

Table 1: Useful quantiles of the standard normal distribution

α	0.4	0.3	0.2	0.1	0.05	0.025	0.020	0.010	0.005	0.001
z_α	0.253	0.524	0.842	1.282	1.645	1.96	2.054	2.326	2.576	3.090
$z_{\alpha/2}$	0.842	1.036	1.282	1.645	1.96	2.241	2.326	2.576	2.807	3.291

Table 2: Useful percentiles of the Chi-square distribution $P[X \le x]$ where X is a chi-square random variable with degrees of freedom r

r	0.010	25.000	0.050	0.100	0.500	0.900	0.950	0.975	0.990
1	0.000	0.001	0.004	0.016	0.455	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	1.386	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	2.366	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	3.357	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	4.351	9.236	11.070	12.833	15.086
6	0.872	1.237	1.635	2.204	5.348	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	6.346	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	7.344	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	8.343	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	9.342	15.987	18.307	20.483	23.209

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3. (30%) Let $Y_{i,} i = 1,2,..,n,$ be i.i.d. discrete uniform distribution on $\{1,2,3,4\}$. And let

$$X_{ki} = \begin{cases} 1, \text{if} & Y_i = k \\ 0, \text{if} & Y_i \neq k \end{cases} \quad \text{for } k = 1,2,3,4.$$

   (a) (5%) Suppose that $\tilde{X}_k = \sum_{i=1}^{n} X_{ki}$. Find the mean and variance of $\tilde{X}_k$.

   (b) (5%) The correlation of $\tilde{X}_1$ and $\tilde{X}_4$ is negative. Give a reason to explain this.

   (c) (10%) Find $\mathrm{Cov}(\tilde{X}_1, \tilde{X}_4)$.

   (d) (10%) Let $Z_1 = \tilde{X}_1 + \tilde{X}_2, Z_2 = \tilde{X}_3$ and $Z_3 = \tilde{X}_4$. Find the joint distribution of (Z_1, Z_2, Z_3).

4. (20%) Let X and Y be independent, standard normal variables.

   (a) (10%) Find the joint distribution of (U, V), where $U = X/_Y$ and $V = |Y|$.

   (b) (2%) Find the sample space of (U, V).

   (c) (8%) Compute the distribution of U.

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