

國立臺北大學 107 學年度碩士班一般入學考試試題

系（所）組別：統計學系

科目：基礎數學

第 1 頁 共 1 頁

☐可 ☒不可使用計算機

本考試不可使用電子計算機。請務必依題號順序作答。配分註記於各子題中。

一、Calculus(50%)。(1)-(6)小題為計算證明題，請清楚完整作答始得完全分數；(7)小題為是非題僅需回答 T 或 F。

- (1). (8%)Find the second difference for the function $f''(x)$ for $f(x) = 0$, if $x=0$; $f(x) = \frac{1-\cos x}{x}$, if $x \neq 0$.
- (2). (8%)Find the limit for $f(x) = \lim_{x \rightarrow \infty} \frac{x^k + 5\cos(x^2)}{e^x}$, where k is some positive finite constant.
- (3). (8 %)Find the integral $\int \frac{2}{x+\sqrt{1+x+x^2}} dx$.
- (4). (8 %)Find the definite integral $\iint_R \ln(x^2 + y^2) dA$, $R = \{(x, y) | 0 \leq x^2 + y^2 \leq 9, x \geq 0, y \geq 0\}$.
- (5). (8%)Prove or disprove whether $f(x, y) = \frac{2x^2y}{x^2+y^2}$, if $(x, y) \neq (0, 0)$, and $f(0, 0)=0$ is differentiable at $(0, 0)$ or not?
- (6). (8%)Find the limit $\lim_{n \rightarrow \infty} \sum_{k=1}^n \ln \sqrt[n]{1 + \frac{2k}{n}}$.
- (7). (T/F; 2%) The limit $\lim_{x \rightarrow 0} f(x)$ does not exist for $f(x) = x^2$, if $x \in \text{rational}$; $f(x) = x^4$, if $x \in \text{irrational}$.

二、所有題目請敘述計算過程，無計算過程不給分

(1). Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

- (a) Write down the characteristic polynomial of A and use it to find the eigenvalues. (5%)
- (b) Find the eigenspaces of A . (10%)
- (c) Orthogonally diagonalize the matrix A . (You need to find out an orthogonal matrix P and a diagonal matrix D such that $P^T A P = D$.) (10%)

- (2). Let B, C be two bases for P_2 . If $B = \{x, 1+x, 1-x+x^2\}$ and the change-of-basis matrix from B to C is

$$P_{C \leftarrow B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ -1 & 1 & 1 \end{bmatrix},$$

please find C . (10%)

- (3). Let $S = \{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ be a basis for $\mathbb{R}^n$ and A be a $n \times n$ nonsingular matrix.

- (a) Please show that $\{A\vec{v}_1, A\vec{v}_2, \dots, A\vec{v}_n\}$ is also a basis for $\mathbb{R}^n$. (12%)
- (b) Is (a) still true if A is singular? If not, please give a counter example. (3%)

試題隨卷繳交