國立臺北大學 107 學年度碩士班一般入學考試試題

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Notes: Please round to the nearest hundredths (the 2nd decimal) in your calculation. In making statistical inferences, clearly specify the hypothesis, test statistics, decision rules, and the conclusions derived from your tests. If needed, use the most appropriate table to find the critical values.

I. (20%) The Heart Association of Taiwan claims that only 10% of Taiwan adults over 25 can pass the fitness test proposed by the Ministry of Health and Welfare. Suppose four adults are randomly selected and each is given the fitness test. Let x represents the number of four adults who pass the fitness test.

(a) Which type of probability distribution can be used to describe the random variable x?
(b) If the Heart Association's claim is true, calculate the mean and standard deviation for x.
(c) The ministry plans to conduct the trail 36 times to observe the value of x. In each trial, they will randomly select four individuals and observe the number of adults who pass the test. Assume again that the Heart Association's claim is true, find the mean and standard deviation for the sample mean, $\bar{x}$.
(d) Again assume that the Heart Association's claim to be valid. If the ministry wants to ensure that the sample mean of the experiments to be within 0.01 of its true value with 99% confidence, what sample size should be specified for the experiment?

II. (15%) Public opinion polls are conducted regularly to estimate the percentage of Taiwan citizens who approve or disapprove of the way that the President is handling his or her job. Suppose 1,000 people were randomly surveyed and 363 of the respondents answered that they approve the President's job performance.

(a) Estimate the fraction of citizens who approve the President's job performance with 95% confidence.
(b) Since the midterm election is approaching, the polling firm decides to conduct another survey and they want the sampling error to be within 1 percentage point with 90% confidence. Use their previous estimate as the planning value, how many people should the firm interview for this survey?

III. (20%) A water processing plant monitors the quality of their drinking water and has to ensure that the pH of their water to be 8.5. Suppose they collect 16 water samples, and the sample's mean pH and its standard deviation is 8.42 and 0.16, respectively. (i.e. $\bar{x} = 8.42, s = 0.16$.)

(a) The plant will adjust its process if there is evidence indicating that the pH level in their water differs from their target level. Assume the manager is only willing to risk a Type I error with 5% probability, does this sample provide sufficient evidence for the plant to adjust its treatment process? Explain your answer.
(b) If the water's actual pH level is 8.38, what is the power from the test that you have conducted in the previous question? Explain your answer and interpret your result in this context.

IV. (20%) An economist has run a between-subjects experiment comparing two treatments (Treatment A and Treatment B) against a control condition. Subjects were randomly assigned to the experimental condition and the result is summarized as follows:

	SAMPLE SIZE	SAMPLE MEAN	STD. DEV.
CONTROL	40	21.4	4.5
TREATMENT A	40	16.9	5.5
TREATMENT B	40	19.1	5.8

Overall sample mean ($n = 120$)： 19.13

Based on the information, can the economist conclude that the average scores among the three groups are different at $\alpha = 0.01$ level of significance?

V. (25%) A collector of antiques wants to know the relationship between the auction prices (in dollars, denoted by P) and the following variables:

A: age of the antique (in years)

B: number of bidders

After inputting the data, the collector received the following printout from a statistical software after he estimated the relationship with an ordinary least square model:

Table V-a: Analysis of Variance

Source	Degrees of Freedom	Sum of Squares	Mean Square	F value	p-value
Model	2	4283063	2141531	120.19	<0.0001
Error	29	516727	17818		
Total	31	4799790			

Table V-b: Parameter Estimates (Dependent Variable: P)

Variable	Estimate	Std. Error
Constant	-1338.95	173.81
Age	12.74	0.90
Bidders	85.95	8.73

(a) Does the mean price of an antique increase as the number of bidders increases? Conduct a hypothesis test ($\alpha = 0.05$) to support your answer.

(b) Calculate the model's R^2 and adjusted R^2. Interpret both numbers in the context of this question.

(c) What conclusion can we derive from the F-value in Table V-a? State any assumptions for your conclusions to be valid, if necessary.

(d) If the age of an antique was found to be *ten years more* than it is expected, how would this information affect its price based on the estimated model? Construct a 90% confidence interval for your answer.

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Table 1: Standard Normal Probability Table

Entries in the table show the upper tail probability, i.e, $\Pr(z \geq Z)$

Z	+0	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46020	0.45620	0.45224	0.44828	0.44433	0.44034	0.43640	0.43251	0.42858	0.42465
0.2	0.42070	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08692	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100

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Table 2: Student's t-distribution table

Entries in the table give t_α values, where α is the probability in the upper tail of the *t*-distribution

Degrees of Freedom	Upper Tail Probability α					
	0.2	**0.1**	**0.05**	**0.025**	**0.01**	**0.005**
1	1.376	3.078	6.314	12.706	31.821	63.657
2	1.061	1.886	2.920	4.303	6.965	9.925
3	0.978	1.638	2.353	3.182	4.541	5.841
4	0.941	1.533	2.132	2.776	3.747	4.604
5	0.920	1.476	2.015	2.571	3.365	4.032
6	0.906	1.440	1.943	2.447	3.143	3.707
7	0.896	1.415	1.895	2.365	2.998	3.499
8	0.889	1.397	1.860	2.306	2.896	3.355
9	0.883	1.383	1.833	2.262	2.821	3.250
10	0.879	1.372	1.812	2.228	2.764	3.169
11	0.876	1.363	1.796	2.201	2.718	3.106
12	0.873	1.356	1.782	2.179	2.681	3.055
13	0.870	1.350	1.771	2.160	2.650	3.012
14	0.868	1.345	1.761	2.145	2.624	2.977
15	0.866	1.341	1.753	2.131	2.602	2.947
16	0.865	1.337	1.746	2.120	2.583	2.921
17	0.863	1.333	1.740	2.110	2.567	2.898
18	0.862	1.330	1.734	2.101	2.552	2.878
19	0.861	1.328	1.729	2.093	2.539	2.861
20	0.860	1.325	1.725	2.086	2.528	2.845
21	0.859	1.323	1.721	2.080	2.518	2.831
22	0.858	1.321	1.717	2.074	2.508	2.819
23	0.858	1.319	1.714	2.069	2.500	2.807
24	0.857	1.318	1.711	2.064	2.492	2.797
25	0.856	1.316	1.708	2.060	2.485	2.787
26	0.856	1.315	1.706	2.056	2.479	2.779
27	0.855	1.314	1.703	2.052	2.473	2.771
28	0.855	1.313	1.701	2.048	2.467	2.763
29	0.854	1.311	1.699	2.045	2.462	2.756
30	0.854	1.310	1.697	2.042	2.457	2.750
40	0.851	1.303	1.684	2.021	2.423	2.704
50	0.849	1.299	1.676	2.009	2.403	2.678
60	0.848	1.296	1.671	2.000	2.390	2.660
80	0.846	1.292	1.664	1.990	2.374	2.639
100	0.845	1.290	1.660	1.984	2.364	2.626
120	0.845	1.289	1.658	1.980	2.358	2.617
∞	0.842	1.282	1.645	1.960	2.326	2.576

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Table 3: F Distribution

Entries in the table give F_α values, where α is the probability in the upper tail of the F distribution

$\alpha = 0.01$	Numerator Degrees of Freedom												
Denominator Degrees of Freedom	1	2	3	4	5	6	7	8	9	10	20	30	40
35	7.419	5.268	4.396	3.908	3.592	3.368	3.200	3.069	2.963	2.876	2.445	2.281	2.193
36	7.396	5.248	4.377	3.890	3.574	3.351	3.183	3.052	2.946	2.859	2.428	2.263	2.175
37	7.373	5.229	4.360	3.873	3.558	3.334	3.167	3.036	2.930	2.843	2.412	2.247	2.159
38	7.353	5.211	4.343	3.858	3.542	3.319	3.152	3.021	2.915	2.828	2.397	2.232	2.143
39	7.333	5.194	4.327	3.843	3.528	3.305	3.137	3.006	2.901	2.814	2.382	2.217	2.128
40	7.314	5.179	4.313	3.828	3.514	3.291	3.124	2.993	2.888	2.801	2.369	2.203	2.114
41	7.296	5.163	4.299	3.815	3.501	3.278	3.111	2.980	2.875	2.788	2.356	2.190	2.101
42	7.280	5.149	4.285	3.802	3.488	3.266	3.099	2.968	2.863	2.776	2.344	2.178	2.088
43	7.264	5.136	4.273	3.790	3.476	3.254	3.087	2.957	2.851	2.764	2.332	2.166	2.076
44	7.248	5.123	4.261	3.778	3.465	3.243	3.076	2.946	2.840	2.754	2.321	2.155	2.065
45	7.234	5.110	4.249	3.767	3.454	3.232	3.066	2.935	2.830	2.743	2.311	2.144	2.054
115	6.861	4.795	3.957	3.487	3.181	2.963	2.799	2.670	2.565	2.479	2.042	1.867	1.770
116	6.859	4.793	3.955	3.485	3.179	2.961	2.797	2.668	2.564	2.477	2.040	1.866	1.769
117	6.857	4.791	3.954	3.484	3.178	2.960	2.796	2.667	2.563	2.476	2.039	1.864	1.767
118	6.855	4.790	3.952	3.482	3.176	2.959	2.794	2.666	2.561	2.475	2.037	1.863	1.766
119	6.853	4.788	3.951	3.481	3.175	2.957	2.793	2.664	2.560	2.473	2.036	1.861	1.764
120	6.851	4.787	3.949	3.480	3.174	2.956	2.792	2.663	2.559	2.472	2.035	1.860	1.763
121	6.849	4.785	3.948	3.478	3.172	2.955	2.790	2.662	2.557	2.471	2.033	1.859	1.761
122	6.847	4.783	3.946	3.477	3.171	2.953	2.789	2.660	2.556	2.470	2.032	1.857	1.760
123	6.846	4.782	3.945	3.476	3.170	2.952	2.788	2.659	2.555	2.468	2.031	1.856	1.759
124	6.844	4.781	3.944	3.474	3.168	2.951	2.787	2.658	2.554	2.467	2.029	1.855	1.757
125	6.842	4.779	3.942	3.473	3.167	2.950	2.786	2.657	2.552	2.466	2.028	1.853	1.756

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$\alpha = 0.005$

Denominator Degrees of Freedom	Numerator Degrees of Freedom 1	2	3	4	5	6	7	8	9	10	20	30	40
35	8.976	6.188	5.086	4.479	4.088	3.812	3.607	3.447	3.318	3.212	2.693	2.50	2.39
36	8.943	6.161	5.062	4.455	4.065	3.790	3.585	3.425	3.296	3.191	2.672	2.475	2.371
37	8.912	6.135	5.038	4.433	4.043	3.769	3.564	3.404	3.276	3.171	2.652	2.455	2.350
38	8.882	6.111	5.016	4.412	4.023	3.749	3.545	3.385	3.257	3.152	2.633	2.436	2.331
39	8.854	6.088	4.995	4.392	4.004	3.731	3.526	3.367	3.239	3.134	2.615	2.418	2.313
40	8.828	6.066	4.976	4.374	3.986	3.713	3.509	3.350	3.222	3.117	2.598	2.401	2.296
41	8.803	6.046	4.957	4.356	3.969	3.696	3.492	3.334	3.206	3.101	2.583	2.385	2.280
42	8.779	6.027	4.940	4.339	3.953	3.680	3.477	3.318	3.191	3.086	2.567	2.370	2.264
43	8.757	6.008	4.923	4.324	3.937	3.665	3.462	3.304	3.176	3.071	2.553	2.356	2.250
44	8.735	5.991	4.907	4.308	3.923	3.651	3.448	3.290	3.162	3.057	2.540	2.342	2.236
45	8.715	5.974	4.892	4.294	3.909	3.638	3.435	3.276	3.149	3.044	2.527	2.329	2.222
115	8.192	5.550	4.507	3.930	3.557	3.294	3.096	2.941	2.817	2.714	2.197	1.993	1.880
116	8.189	5.548	4.505	3.928	3.555	3.292	3.094	2.940	2.815	2.712	2.195	1.991	1.878
117	8.187	5.546	4.503	3.926	3.553	3.290	3.092	2.938	2.813	2.710	2.193	1.989	1.876
118	8.184	5.544	4.501	3.924	3.552	3.288	3.091	2.936	2.812	2.708	2.191	1.987	1.874
119	8.181	5.541	4.499	3.922	3.550	3.287	3.089	2.935	2.810	2.707	2.190	1.986	1.873
120	8.179	5.539	4.497	3.921	3.548	3.285	3.087	2.933	2.808	2.705	2.188	1.984	1.871
121	8.176	5.537	4.495	3.919	3.547	3.283	3.086	2.931	2.807	2.704	2.187	1.982	1.869
122	8.174	5.535	4.493	3.917	3.545	3.282	3.084	2.930	2.805	2.702	2.185	1.981	1.868
123	8.171	5.533	4.492	3.915	3.543	3.280	3.083	2.928	2.804	2.701	2.183	1.979	1.866
124	8.169	5.531	4.490	3.914	3.542	3.278	3.081	2.927	2.802	2.699	2.182	1.978	1.864
125	8.167	5.529	4.488	3.912	3.540	3.277	3.080	2.925	2.801	2.698	2.180	1.976	1.863

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