

國立臺北大學 107 學年度碩士班一般入學考試試題

系（所）組別：財政學系

科 目：微積分

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☐可 ☒不可 使用計算機

1. (10%) (a) What is the definition of the *Definite Integral* of a function $f(x)$ over $[a, b]$? (b) What is the definition of *The Fundamental Theorem of Calculus*?
2. (10%) Find the limits: (a) $\lim_{x \rightarrow 1} \frac{x^{50} - 1}{x - 1}$. (b) $\lim_{x \rightarrow \infty} \frac{x + \sin x + 2\sqrt{x}}{x + \sin x}$.
3. (15%) Let $f(x) = |x|$. (a) Show that $f(x)$ is continuous at $x = 0$. (b) Show that $f(x)$ is not differentiable at $x = 0$.
4. (10%) Let $f(x) = \frac{x-1}{x+1}$. Use the definition of the derivative of a function to find the instantaneous rate of change of $f(x)$ at $x = -1/2$.
5. (10%) Find dy/dx where $y = \int_2^{x^2} \sin(t^3) dt$.
6. (15%) Sketch the graph of the function $f(x) = \frac{x^2 - 9}{x^2 + 1}$. ($1/\sqrt{3} \approx 0.58$) (identify relative extrema, inflection points, concavity, and asymptotes if any)
7. (10%) Find the values of p for which the integral $\int_2^{\infty} \frac{dx}{x(\ln x)^p}$ converges.
8. (10%) Investigate the convergence of the series.
(a) $\sum_{n=1}^{\infty} \frac{4^n n! n!}{(2n)!}$. (b) $\sum_{n=0}^{\infty} ar^n$, where a is a nonzero real number.
9. (10%) Find the volume of the solid that lies under the surface $z = xy^2$ and above the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$.