東海大學 104 學年度碩士班招生考試試題

(10%) **1.** Suppose that the joint density of X and Y is given by

$$f(x,y) = \frac{e^{-x/y}e^{-y}}{y}, \qquad 0 < x < \infty, 0 < y < \infty.$$

Compute $E(X|Y=y)$.

2. Compute the following.

(5%) (a) Find the coefficient of x^2y^3 in the expansion of $(2x+3y)^5$.

(5%) (b) Two cards are randomly selected from an ordinary playing deck. What is the probability that they form a blackjack? That is, what is the probability that one of the cards is an ace and the other one is either a ten, a jack, a queen, or a king?

(10%) **3.** Let the random variable X have the pdf

$$f(x) = \frac{2}{\sqrt{2\pi}}e^{-x^2/2}, \qquad 0 < x < \infty, \qquad \text{zero elsewhere.}$$

Find the mean and variance of X.

(10%) **4.** Let X be a random variable with pdf ($\alpha = 50$ is known)

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}, \quad \alpha > 0, \beta > 0, x > 0.$$

If a random sample of size $n = 14$ has the sample mean 2.3. Find the maximum likelihood estimate of β.

(10%) **5.** Let X, Y, Z be independent and uniformly distributed over (0,1). Compute $P(X \geq YZ)$.

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(10%) **6.** Let X_1, X_2, X_3, X_4 be i.i.d. uniform(0,1) distribution. If $X_{(1)} < X_{(2)} < X_{(3)} < X_{(4)}$ are the order statistics. Let $R = X_{(4)} - X_{(1)}$. Find $P(R \geq 1/2)$.

(10%) **7.** Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables from exponential distribution with pdf $f(x) = \theta e^{-\theta x}$, $x > 0$, $\theta > 0$. Apply the Rao-Blackwell Theorem to find an uniformly minimum variance unbiased estimator (UMVUE) of θ.

(10%) **8.** Let $X_1, X_2, \ldots, X_n$ be i.i.d. Poisson distribution with parameter one. Let $Z = X_1 + X_2 + \ldots + X_n$. Find the limit distribution of $\sqrt{Z} - \sqrt{n}$.

9. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(0, \theta)$, $0 < \theta < \infty$.

(5%) (a) Find the Fisher information $I(\theta)$.

(5%) (b) Show that the maximum likelihood estimator (MLE) of θ is an efficient estimator of θ.

(10%) **10.** Let X be a random variable whose pmf under H_0 and H_1 is given by

x	1	2	3	4	5	6	7
$f(x\|H_0)$	.01	.01	.01	.01	.01	.01	.94
$f(x\|H_1)$	.06	.05	.04	.03	.02	.01	.79

Use the Neyman-Pearson Lemma to find the most powerful test for H_0 versus H_1 with size $\alpha = .04$. Compute the probability of Type II Error for this test.