# 國立高雄大學一百學年度研究所碩士班招生考試試題

科目：數理統計
考試時間：100 分鐘

系所：
統計學研究所(統計組)
本科原始成績：100 分

是否使用計算機：否

1. (15%) Let $(X, Y)$ be random variables with joint density

$$f(x,y) = 360xy^2(1-x-y),\ x>0,\ y>0,\ x+y \le 1.$$

   (a) Find the conditional density function of $Y$ given $X = x$, $f(y|x)$.

   (b) Compute $\mathrm{Var}(Y|x)$.

2. (15%) Let $X_1, \cdots, X_n$ be a random sample from the density

$$f(x) = \lambda e^{-\lambda x},\ x>0,\ \lambda>0.$$

   Let $Z_n = X_{(1)}$ be the smallest order statistics.

   (a) Find the limiting distribution of $Z_n$.

   (b) Find the limiting distribution of $nZ_n$.

3. (15%) Let $X_1, \cdots, X_n$ be a random sample from the uniform distribution on the interval $(\theta, \theta+1)$, $-\infty < \theta < \infty$. Find a minimal sufficient statistic for $\theta$. Is it complete? Why or why not?

4. (15%) Let $X_1, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$ distribution, $\mu \in R$, $\sigma^2 > 0$, where $\mu$ and $\sigma^2$ are both unknown. Find the UMVUE (uniformly minimum variance unbiased estimator) of $\sigma^2$. Does its variance attain the CRLB (Cramér-Rao lower bound)?

5. (20%) Let $X_1, \cdots, X_n$ be a random sample from the Poisson distribution with intensity rate $\lambda > 0$. Find the MLE (maximum likelihood estimator) and the UMVUE of $P(X_1 = 0)$.

6. (20%) Let $X_1, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$ distribution, where $\sigma^2$ is unknown. Consider testing $H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$.

   (a) Does a UMP (uniformly most powerful) test exist? Why or why not?

   (b) Show that the test that rejects $H_0$ when

$$|\overline{X} - \mu_0| > t_{n-1,\alpha/2}\sqrt{S^2/n}$$

   can be derived as an LRT (likelihood ratio test), where $t_{n-1,\alpha/2}$ satisfies $P(T_{n-1} \ge t_{n-1,\alpha/2}) = \alpha/2$ with $T_{n-1}$ following the $t$ distribution with $n-1$ degrees of freedom, $\overline{X}$ is the sample mean and $S^2$ is the sample variance.