

東海大學 101 學年度碩士班招生入學考試試題

考試科目：微積分 E

應考系所：統計系乙組

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(如有缺損或印刷不清者，應即舉手請監試人員處理)

1. (10%) For the function $f(x) = \sqrt{x^2 - 4}$, determine (a) $f(-3)$ and (b) the domain of the function $f(x)$.

2. (24%) Evaluate the following limits.

(a) $\lim_{x \rightarrow 2} \frac{2x^2 - 4x}{x - 2}$ (b) $\lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h}$ (c) $\lim_{x \rightarrow \infty} \frac{x^2}{1+x+2x^2}$.

3. (10%) Consider the function

$$f(x) = \begin{cases} 2x + 3, & \text{if } x < 1; \\ Ax - 1, & \text{if } x \geq 1. \end{cases}$$

Find the value of the constant A that makes the function $f(x)$ continuous for all x .

4. (12%) The first derivative of a certain function is $f'(x) = x(x - 1)^2$.

(a) On what intervals is f increasing? Decreasing?

(b) On what intervals is the graph of f concave up? Concave down?

5. (24%) Evaluate the following integrals.

(a) $\int (e^{2x} - \frac{2}{x}) dx$ (b) $\int x \ln x dx$ (c) $\int_0^1 \int_0^y y^2 e^{xy} dx dy$.

6. (8%) Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{3^{n+1}}$.

7. (12%) Find the second Taylor polynomial for $f(x) = \sqrt{x}$ at $x = 4$ (ie. $P_2(x) = f(4) + f'(4)(x - 4) + \frac{1}{2}f''(4)(x - 4)^2$), and use it (ie. $P_2(4.2)$) to approximate $\sqrt{4.2}$.