東海大學 101 學年度碩士班招生入學考試試題

考試科目：統計學F　　　　應考系所：統計系甲組

本試題共2頁：第1頁　　（如有缺損或印刷不清者，應即舉手請監試人員處理）

10 points for each problem

1. A box of candy hearts contains 52 hearts, of which 19 are white, 10 are tan, 7 are pink, 3 are purple, 5 are yellow, 2 are orange, and 6 are greens. If you select 9 pieces of candy randomly from the box, without replacement, calculate the probability that three of the hearts are white.

2. Suppose that the random variables $X_1, X_2, \ldots, X_k$ are independent and that each X_i has a Poisson distribution with mean λ_i $(i = 1, 2, \ldots, k)$. Derive that for any fixed integer n, the conditional distribution of the random vector $\mathbf{X} = (X_1, X_2, \ldots, X_k)$ given that $\sum_{i=1}^{k} X_i = n$ is a multinomial distribution. Also need to provide the parameters in terms of λ_i $(i = 1, 2, \ldots, k)$.

3. Suppose that the joint probability density function of two random variables X and Y is as follows:

$$f(x, y) = \begin{cases} c(x + y^2), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the conditional probability density function of X given Y and find $P(X < \frac{1}{2} | y = \frac{1}{2})$.

4. Suppose $X_1, X_2, \ldots, X_n$ is an independent random sample iid from normal distribution $N(\mu, \sigma^2)$. Derive the maximum likelihood estimates of μ and σ^2.

5. Consider a probability density function as follows:

$$f(x) = \frac{1}{\theta} x^{(1-\theta)/\theta},\ 0 < x < 1,\ 0 < \theta < \infty$$

Suppose $X_1, X_2, \ldots, X_n$ is an independent random sample with density function $f(x)$, where θ is unknown. Find an unbiased estimate $\hat{\theta}$ of θ where all the values $X_1, X_2, \ldots, X_n$ are used to calculate this estimate $\hat{\theta}$. (Hint: MLE)

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6. Suppose that $X_1, X_2, \ldots, X_n$ is an indepdent random sample from a uniform distribution on the interval $(0,1)$ and let $Y_1 = \min(X_1, X_2, \ldots, X_n)$ and $Y_n = \max(X_1, X_2, \ldots, X_n)$. Determine the value of $P(Y_1 \le 0.1 \text{ and } Y_n \le 0.8)$.

7. Suppose that $X_1, X_2, \ldots, X_m$ is an independent random sample from $N(\mu_x, \sigma^2)$ and $Y_1, Y_2, \ldots, Y_n$ is an independent random sample from $N(\mu_y, \sigma^2)$. Construct a $100(1-\alpha)\%$ confidence interval for $\mu_x - \mu_y$, where σ^2 is unknown.

8. Suppose that we have two independent random samples: $X_1, X_2, \ldots, X_m$ are from exponential distribution with mean θ and $Y_1, Y_2, \ldots, Y_n$ are from exponential distribution with mean μ respectively. Both $\theta \neq 0$ and $\mu \neq 0$. To test $H_0 : \theta = \mu$ against $H_1 : \theta \neq \mu$. Determine the likelihood ratio test to perform the hypothesis test.

9. Suppose that $X_1, X_2, \ldots, X_n$ are IID with a beta distribution $beta(\alpha, 2)$.

$$f(x) = \frac{\Gamma(\alpha+2)}{\Gamma(\alpha)\Gamma(2)} x^{\alpha-1}(1-x),\ 0 < x < 1$$

Let $\overline{X}$ be the sample mean. Find the asymptotic normal distribution of $\overline{X}$.

10. A group of 500 individuals were asked to cast their votes regarding a particular issue of the Equal Rights Amendment. The following contingency table shows the results of the votes:

Gender	Favor	Undecided	Oppose	TOTAL
Female	180	80	40	300
Male	150	20	30	200
TOTAL	330	100	70	500

Find expected frequency for each cell under independence assumption.