國立東華大學招生考試試題

招生學年度	101	招生類別	碩士班
系所班別	應用數學系統計碩士班		
科目	基礎數學		
注意事項	本考科禁止使用掌上型計算機；含微積分及線性代數		

1. (15%) Find
$$\int \frac{dx}{x^4+4}.$$

2. (15%) Find
$$\iint_{\Omega} \frac{1}{(1+x^2+y^2)^{\frac{3}{2}}}dxdy,$$
where $\Omega = \{(x,y) : x \le 1,\, y \ge 0 \text{ and } x \ge y\}$. (Hint: $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + C$.)

3. (15%)
   (a) (3%) State the Rolle's Theorem.
   (b) (7%) Prove that if f and g are differentiable on (a,b), continuous on $[a,b]$, and g' is never 0 in (a,b), then there exists a number c in (a,b), such that
   $$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}.$$
   (Hint: Consider $G(x) = [g(b)-g(a)][f(x)-f(a)]-[g(x)-g(a)][f(b)-f(a)]$.)
   (c) (5%) State and prove the Mean-Value Theorem.

4. (15%) Solve the linear differential system
$$\begin{aligned} x_1' &= x_1 - x_2 - x_3 \\ x_2' &= -x_1 + x_2 - x_3 \\ x_3' &= -x_1 - x_2 + x_3. \end{aligned}$$

5. (15%) Consider the vector space P_2 of polynomials of degree at most 2. Let $T : P_2 \to P_2$ be the linear transformation such that $T(1) = 3+2x+x^2$, $T(x) = 2$, $T(x^2) = 2x^2$. Find $T^{100}(x+2)$.

6. (25%) Let W be a subspace of $\mathbb{R}^5$, $\beta = \{(1,-1,1,0,1), (3,-1,2,1,2), (8,-9,5,-11,-2)\}$ be a basis for W.
   (a) (6%) Find an orthonormal basis for W that contains $(\frac{1}{2}, \frac{-1}{2}, \frac{1}{2}, 0, \frac{1}{2})$.
   (b) (6%) Find a basis for $W^{\perp}$. ($W^{\perp}$ is the orthogonal complement of W)
   (c) (6%) Find the projection of $b = (4,0,-2,-1,2)$ on W and the distance from the point $p = (4,-1,-2,0,1)$ in $\mathbb{R}^5$ to the subspace W.
   (d) (7%) Let P be the projection matrix for W.
      i. (2%) Find all eigenvalues of P.
      ii. (5%) Show that P is diagonalizable.