國立中央大學101學年度碩士班考試入學試題卷

所別：統計研究所碩士班 不分組(一般生)　科目：數理統計　共 2 頁 第 1 頁
統計研究所碩士班 不分組(在職生)

本科考試可使用計算器，廠牌、功能不拘　*請在試卷答案卷（卡）內作答

1. Let $X_1, X_2, \ldots, X_m$ be a random sample from the normal distribution $N(\mu, \sigma^2)$. Also, let $Y_1, Y_2, \ldots, Y_n$ be a random sample from $N(2\mu, 4\sigma^2)$. Consider $g(\bar{X}, \bar{Y}) = a\bar{X} + b\bar{Y}$, where $\bar{X}$ and $\bar{Y}$ are the sample means of the X and Y samples, respectively.
   (a) Find the values of a and b such that $g(\bar{X}, \bar{Y})$ is an unbiased estimator of μ with minimum variance. (10%)
   (b) Find an estimator of the variance of the $g(\bar{X}, \bar{Y})$ in (a). (8%)
   (c) Construct a 100(1-α)% confidence interval for μ based on the $g(\bar{X}, \bar{Y})$ in (a). (6%)

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from the normal distribution $N(\mu, \sigma^2)$.
   (a) Find the first and third quartiles (25th and 75th percentiles) of $N(\mu, \sigma^2)$, denoted by q_1 and q_3, respectively, and hence the inter-quartile range $IQR = q_3 - q_1$. (10%)
   (b) Find the maximum likelihood estimator of the IQR in (a) based on $X_1, X_2, \ldots, X_n$. (6%)

   [Hint: The 75th percentile of a standard normal distribution is z(0.75)=0.674].

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from the exponential distribution with probability density function (pdf)

$$f(x;\theta) = \theta\, exp(-\theta x), x > 0; = 0, \text{ otherwise.}$$

   (a) Verify that $2\theta\sum_{i=1}^{n} X_i$ has a Chi-square distribution with degrees of freedom $2n$, denoted by χ^2_{2n}. (10%)
   (b) Derive a 100(1-α)% confidence interval for $P(X > x)$. (10%)

注意：背面有試題

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4. Let $X_1, X_2, \ldots, X_n$ be serially correlated random variables that satisfy

$$X_i = \theta X_{i-1} + \varepsilon_i,\ i = 1,\ \ldots, n,$$

where $X_0 = 0$ and the ε_i are independent $N(0, \sigma^2)$ random variables.

(a) Find the maximum likelihood estimators of θ and σ^2. (10%)

(b) Find the likelihood ratio test for $H_0 : \theta = 0$ against $H_1 : \theta \neq 0$. (10%)

5. (a) Let X_1 and X_2 be independent random variables from a Bernoulli trial in which the probability of success may take a value of θ_1 or θ_2. Assume that

$$f(0|\theta_1)=0.9,\ f(1|\theta_1)=0.1,\ f(0|\theta_2)=0.2 \text{ and } f(1|\theta_2)=0.8.$$

Let the associated prior distribution be $\pi(\theta_1) = 0.25$ and $\pi(\theta_2) = 0.75$. Find the posterior distribution of θ given $X_1 + X_2 = 1$ and test for $H_0 : \theta = \theta_1$ against $H_1 : \theta = \theta_2$ based on the posterior distribution. (10%)

(b) Let X be the number of failures before the first success in a sequence of Bernoulli trials with probability of success θ. Suppose that $\pi(\theta) = 1/3$ for $\theta = 0.25, 0.5, 0.75$ and 0, otherwise. Find the posterior distribution of θ given X =2 and comment the most probable value of θ. (10%)

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