

系所組別： 交通管理科學系乙、丙組

考試科目： 微積分

考試日期： 0225 · 節次： 2

1. Evaluate the following limits

(a) (5 points) $\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x - 7}$,

(b) (5 points) $\lim_{x \rightarrow 0^+} x^{1/x}$.

2. Evaluate the following integrals

(a) (5 points) $\int_1^4 \sqrt{x} \ln x \, dx$,

(c) (5 points) $\int_0^2 \frac{x}{(1+x^2)^2} \, dx$,

(b) (5 points) $\int_0^1 \int_x^1 e^{x/y} \, dy \, dx$,

(d) (10 points) $\int \int_{x^2+xy+y^2 \leq 1} e^{-(x^2+xy+y^2)} \, dx \, dy$.

3. (5 points) Suppose that f satisfies the equation $f(x+y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers x and y . Suppose further that

$$\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1.$$

Find $f'(x)$.

4. (10 points) Set up TWO different definite integrals expressing the volume of the solid obtained by revolving the region bounded by the curves $y = e^x$, $y = 2$, and $x = 0$ about the line $y = -1$. Do not solve the integrals.

5. (8 points) If $\int_0^x f(t) \, dt = e^{2x} \cos x + c$, find the value of the constant c and the function $f(t)$.

6. (10 points) Let f be a differentiable function with derivative f' . A table of values for f and f' is given below.

t	0	3	6	9
$f(t)$	1	2	7	5
$f'(t)$	1	4	-1	-2

Find the value of $\int_0^1 f'(3t) \, dt$.

7. (a) (10 points) By completing the square, show that

$$\int_0^{1/2} \frac{dx}{x^2 - x + 1} = \frac{\pi}{3\sqrt{3}}.$$

(Hints: Write $x^2 - x + 1 = (x - a)^2 + b$. Recall that $\frac{d}{dx} \arctan x = \frac{1}{x^2+1}$)

(b) (10 points) Rewrite the integral in part (a) by express $1/(x^2 - x + 1)$ as the sum of two power series (Hint: $x^3 + 1 = (x + 1)(x^2 - x + 1)$). Prove the following formula for π :

$$\pi = \frac{3\sqrt{3}}{4} \sum_{n=0}^{\infty} \frac{(-1)^n}{8^n} \left(\frac{2}{3n+1} + \frac{1}{3n+2} \right).$$

8. (12 points) Let $f(x, y) = \frac{x^3}{3} + \frac{y^3}{3} - 2x^2 - y^2$. Find all of the critical points of f , and classify each critical point as a local maximum, local minimum, or saddle point.

This exam has 8 questions, for a total of 100 points.