注意事項：

1. 本試卷共有 8 大題，每題配分標註於題號後，合計 100 分。
2. 請<u>依序作答</u>，且必須寫出推導與計算過程，否則不予計分。

[1]. (10%)　Consider a random variable X with the following mixed probability function:

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } 0 \le x < 1 \\ a & \text{if } x = 1 \\ \frac{1}{4} & \text{if } 1 < x \le 2 \end{cases}$$

(1) What is the value of a? (5%)
(2) What is the expected value of $E(X)$? (5%)

[2] (15%)　Consider tossing a fair dice. Let $Y = X_{max} - X_{min}$ in which X_{max} and X_{min} are the largest and the smallest number in three consecutive tosses respectively.
(1) What is the probability of $P(Y = 3)$? (5%)
(2) What is the expected value of $E(Y)$? (10%)

[3]. (10%)　The average content of a bottling process is normally distributed with mean **600** cc and standard deviation of **2**. The customer's specification requirement is 600 ± 5 cc.
(1) What is the defective percentage of the bottling process? (5%)
(2) A quality engineer wants to reduce the defective percentage by reducing the variation. What is the required standard deviation in order to reduce the defective percentage to **1%**? (5%)

[4]. (5%)　Consider the hypothesis test: $H_0 : \mu \ge 150$　vs　$H_a : \mu < 150$. The population standard deviation is known to be **50**. Assume type I error is **0.05**. What is the sample size needed if type II error is **0.10** when the actual population mean is **170**?

[5]. (10%)　A job designer wants to know if the modified method improves the job. The following are the job processing times measured for the original and the modified methods:

Original	109	105	102	112	112	116	102	110	112	110
Modified	107	102	94	108	104	102	96	104	101	109

(1) For $\alpha = 0.01$, conduct the hypothesis test if the times are measured **independently**. (5%)
(2) For $\alpha = 0.01$, conduct the hypothesis test if the times are measured **in pair**. (5%)

[6]. (15%) In 2015, **40%** of the students at a major university were Business majors, **35%** were Engineering majors, and the rest of the students were majoring in other fields. In a sample of **600** students from the same university taken in 2016, **200** were Business majors, **220** were Engineering majors, and the remaining students in the sample were majoring in other fields. Using $\alpha = .05$, test to see if there has been a significant change in the proportions between 2015 and 2016.

[7]. (15%) Eight observations were selected from each of three populations (total of **24** observations), and an analysis of variance was performed on the data. The following are part of the results. 〔3% each〕

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F
Between Treatments	216	_____	_____	_____
Within Treatments (Error)	252	_____	_____	

Complete the **ANOVA table** above. Using $\alpha = .05$, test to see if there is a significant difference among the means of the three populations. State **the null and alternative hypotheses**, and compute the **test statistic** for this test.

[8]. (20%) Shown below is a portion of the computer output for a regression analysis relating sales (y in millions of dollars) and advertising expenditure (x in thousands of dollars). 〔4% each〕
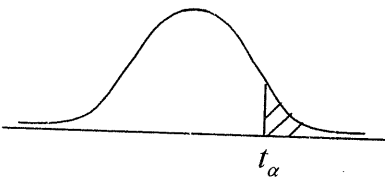
Predictor	Coefficient	Standard Error
Constant	4.00	0.800
x	0.12	0.045

ANOVA

	df	SS
Regression	1	1400
Error	18	3600

(1) What has been the sample size for the above regression analysis?

(2) Perform a t test and determine whether or not advertising and sales are related. Let $\alpha = .05$.

(3) Compute the coefficient of determination.

(4) Interpret the meaning of the value of the coefficient of determination that you found in Part (3). Be very specific.

(5) Use the estimated regression equation and predict sales for an advertising expenditure of **$4000**. Give your answer in dollars.
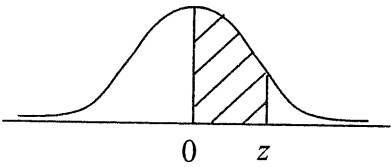
Appendix: Distribution Tables
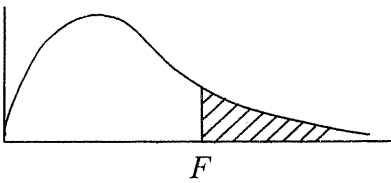
Table 1: *t* Distribution Critical Value Table

$$t_\alpha$$

α value

Degrees of Freedom	0.3	0.2	0.1	0.05	0.025	0.01	0.005	0.001
1	0.727	1.376	3.078	6.314	12.706	31.821	63.657	318.309
2	0.617	1.061	1.886	2.920	4.303	6.965	9.925	22.327
3	0.584	0.978	1.638	2.353	3.182	4.541	5.841	10.215
4	0.569	0.941	1.533	2.132	2.776	3.747	4.604	7.173
5	0.559	0.920	1.476	2.015	2.571	3.365	4.032	5.893
6	0.553	0.906	1.440	1.943	2.447	3.143	3.707	5.208
7	0.549	0.896	1.415	1.895	2.365	2.998	3.499	4.785
8	0.546	0.889	1.397	1.860	2.306	2.896	3.355	4.501
9	0.543	0.883	1.383	1.833	2.262	2.821	3.250	4.297
10	0.542	0.879	1.372	1.812	2.228	2.764	3.169	4.144
11	0.540	0.876	1.363	1.796	2.201	2.718	3.106	4.025
12	0.539	0.873	1.356	1.782	2.179	2.681	3.055	3.930
13	0.538	0.870	1.350	1.771	2.160	2.650	3.012	3.852
14	0.537	0.868	1.345	1.761	2.145	2.624	2.977	3.787
15	0.536	0.866	1.341	1.753	2.131	2.602	2.947	3.733
16	0.535	0.865	1.337	1.746	2.120	2.583	2.921	3.686
17	0.534	0.863	1.333	1.740	2.110	2.567	2.898	3.646
18	0.534	0.862	1.330	1.734	2.101	2.552	2.878	3.610
19	0.533	0.861	1.328	1.729	2.093	2.539	2.861	3.579
20	0.533	0.860	1.325	1.725	2.086	2.528	2.845	3.552
21	0.532	0.859	1.323	1.721	2.080	2.518	2.831	3.527
22	0.532	0.858	1.321	1.717	2.074	2.508	2.819	3.505
23	0.532	0.858	1.319	1.714	2.069	2.500	2.807	3.485
24	0.531	0.857	1.318	1.711	2.064	2.492	2.797	3.467
25	0.531	0.856	1.316	1.708	2.060	2.485	2.787	3.450
26	0.531	0.856	1.315	1.706	2.056	2.479	2.779	3.435
27	0.531	0.855	1.314	1.703	2.052	2.473	2.771	3.421
28	0.530	0.855	1.313	1.701	2.048	2.467	2.763	3.408
29	0.530	0.854	1.311	1.699	2.045	2.462	2.756	3.396
30	0.530	0.854	1.310	1.697	2.042	2.457	2.750	3.385
40	0.529	0.851	1.303	1.684	2.021	2.423	2.704	3.307
50	0.528	0.849	1.299	1.676	2.009	2.403	2.678	3.261
60	0.527	0.848	1.296	1.671	2.000	2.390	2.660	3.232
70	0.527	0.847	1.294	1.667	1.994	2.381	2.648	3.211
80	0.526	0.846	1.292	1.664	1.990	2.374	2.639	3.195
90	0.526	0.846	1.291	1.662	1.987	2.368	2.632	3.183
100	0.526	0.845	1.290	1.660	1.984	2.364	2.626	3.174
120	0.526	0.845	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.092

Table 2: The Standard Normal Distribution

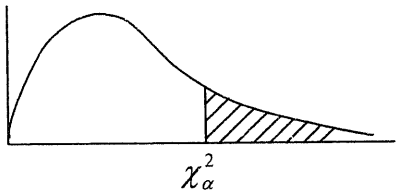| Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995

Table 3: F Distribution Critical Value Table
(α value=0.05)

Denominator degrees of freedom (ddf) / Numerator degrees of freedom (ndf)

ddf \ ndf	1	2	3	4	5	6	7	8	9	10	12	15	20	40	60
1	161.400	199.500	215.700	224.600	230.200	234.000	236.800	238.900	240.500	241.900	243.900	246.000	248.000	251.100	252.200
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385	19.396	19.413	19.429	19.446	19.471	19.479
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786	8.745	8.703	8.660	8.594	8.572
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.912	5.858	5.803	5.717	5.688
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735	4.678	4.619	4.558	4.464	4.431
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.000	3.938	3.874	3.774	3.740
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.575	3.511	3.445	3.340	3.304
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388	3.347	3.284	3.218	3.150	3.043	3.005
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.073	3.006	2.936	2.826	2.787
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	2.913	2.845	2.774	2.661	2.621
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.788	2.719	2.646	2.531	2.490
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687	2.617	2.544	2.426	2.384
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604	2.533	2.459	2.339	2.297
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.534	2.463	2.388	2.266	2.223
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544	2.475	2.403	2.328	2.204	2.160
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.425	2.352	2.276	2.151	2.106
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.381	2.308	2.230	2.104	2.058
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342	2.269	2.191	2.063	2.017
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308	2.234	2.155	2.026	1.980
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.278	2.203	2.124	1.994	1.946
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366	2.321	2.250	2.176	2.096	1.965	1.916
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297	2.226	2.151	2.071	1.938	1.889
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275	2.204	2.128	2.048	1.914	1.865
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255	2.183	2.108	2.027	1.892	1.842
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236	2.165	2.089	2.007	1.872	1.822
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.220	2.148	2.072	1.990	1.853	1.803
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204	2.132	2.056	1.974	1.836	1.785
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190	2.118	2.041	1.959	1.820	1.769
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177	2.104	2.027	1.945	1.806	1.754
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165	2.092	2.015	1.932	1.792	1.740

Table 4: Chi-square Distribution Critical Value Table

α value

Degrees of Freedom	0995	0.99	0.975	0.95	0.9	0.8	0.2	0.1	0.05	0.025	0.01	0.005
1	0.000	0.000	0.001	0.004	0.016	0.064	1.642	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	0.446	3.219	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	1.005	4.642	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	1.649	5.989	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	2.343	7.289	9.236	11.07	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	3.070	8.558	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	3.822	9.803	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	4.594	11.030	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	5.380	12.242	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	6.179	13.442	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	6.989	14.631	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	7.807	15.812	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	8.634	16.985	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	9.467	18.151	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	10.307	19.311	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	11.152	20.465	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	12.002	21.615	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	12.857	22.760	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	13.716	23.900	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	14.578	25.038	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	15.445	26.171	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	16.314	27.301	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	17.187	28.429	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	18.062	29.553	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	18.940	30.675	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	19.820	31.795	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	20.703	32.912	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	21.588	34.027	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	22.475	35.139	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	23.364	36.250	40.256	43.773	46.979	50.892	53.672
31	14.458	15.655	17.539	19.281	21.434	24.255	37.359	41.422	44.985	48.232	52.191	55.003
32	15.134	16.362	18.291	20.072	22.271	25.148	38.466	42.585	46.194	49.480	53.486	56.328
33	15.815	17.074	19.047	20.867	23.110	26.042	39.572	43.745	47.400	50.725	54.776	57.648
34	16.501	17.789	19.806	21.664	23.952	26.938	40.676	44.903	48.602	51.966	56.061	58.964
35	17.192	18.509	20.569	22.465	24.797	27.836	41.778	46.059	49.802	53.203	57.342	60.275
40	20.707	22.164	24.433	26.509	29.051	32.345	47.269	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	41.449	58.164	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	50.641	68.972	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	59.898	79.715	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	69.207	90.405	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	78.558	101.054	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	87.945	111.667	118.498	124.342	129.561	135.807	140.169
110	75.550	78.458	82.867	86.792	91.471	97.362	122.250	129.385	135.480	140.917	147.414	151.948
120	83.852	86.923	91.573	95.705	100.624	106.806	132.806	140.233	146.567	152.211	158.950	163.648