

考 試 科 目	統計學 B	所 別	金融學系 財務工程與金融 科技組	考 試 時 間	2 月 2 日(五)第二節
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1. Consider a probability space (Ω, F, P) with $\Omega = \{1, 2, 3, 4\}$, $F = \sigma(\{1\}, \{2\}, \{3\}, \{4\})$, and

$$P(\{1\}) = \frac{1}{3}, P(\{2\}) = \frac{1}{6}, P(\{3\}) = \frac{1}{4}, P(\{4\}) = \frac{1}{4}$$

Define three random variables, X and Y , by

$$X(1) = 1, X(2) = 1, X(3) = -1, X(4) = -1$$

$$Y(1) = 1, Y(2) = -1, Y(3) = 1, Y(4) = -1$$

- (1) List the sets in $\sigma(X)$. (5%)
 (2) Please find $P(Y|X)$ and $E(Y|X)$. (10%)
 (3) Let $B = \{1, 2, 4\}$. For $A \in F$, find $P(A|B)$. (10%)
2. (1). Please describe the assumptions of the Black-Scholes option pricing formula. (5%)
 (2). Under physical probability measure \mathcal{P} , given the dynamics of the stock price

$$dS_t = \mu S_t dt + \sigma S_t dW_t^{\mathcal{P}} \quad (1)$$

where dS_t denotes the stock change at instantaneous time, μ presents the expected return of the stock at instantaneous time, dt is the instantaneous time, σ means the standard deviation (volatility) of the stock return, and $dW_t^{\mathcal{P}}$ is the change of the Brownian Motion at instantaneous time under \mathcal{P} .

Please derive the Black-Scholes pricing formula at time 0 for European call option with the strike K , the maturity T and the risk-free interest rate r by your known method. (20%)

備

註

- 一、作答於試題上者，不予計分。
 二、試題請隨卷繳交。

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(3). Based on Equation (1), the function $f = \ln S_t$, and Ito's Lemma, find the solution of the stochastic

differential Equation (1) $S_T = S_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W_T^P}$ from 0 to T . (10%)

(4). According to the data of the return at the stock every day, $\{r_1, r_2, \dots, r_n\}$, how to estimate the parameters μ and σ^2 . (10%)

(5). The answer of the problem (2) is the pricing formula of the European call option, where the option price has five parameters, and we have one option market price C , please find which parameter is unknown and obtained by the Black-Scholes option pricing formula and the option market price, and what it is called. (10%)

(6). From the answers of the problem (4) and the problem (5), we obtain two volatilities. Please explain what is the meaning for that two volatilities. (10%)

(7). Under the risk-neutral measure \mathbb{Q} , given the dynamics of the stock price

$$dS_t = rS_t dt + \sigma S_t dW_t^{\mathbb{Q}} \quad (2)$$

where r presents the risk-neutral interest rate at instantaneous time, and $dW_t^{\mathbb{Q}}$ is the change of the Brownian Motion at instantaneous time under the risk-neutral measure \mathbb{Q} .

Please give the meaning of the risk-neutral measure \mathbb{Q} and how to find the change measure by Givsanov Theorem. (10%)

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