國立高雄大學 106 學年度研究所碩士班招生考試試題

科目：統計學
考試時間：100 分鐘

系所：統計學研究所（統計組、風險管理組）

是否使用計算機：否

本科原始成績：100 分

1. (10%) 設X有$U(0,1)$，令$Y=X^r, r>0$，試求 (a) Y之 p.d.f.，(b) 並指出此為那一常見分佈，參數為何？

2. (20%) 設(X,Y)之聯合 p.d.f.為$f(x,y)=360xy^2(1-x-y), x,y \geq 0, x+y \leq 1$。試求 (a) X之邊際分佈；(b) Y|X=x 之條件分佈；(c) E(Y|X=x) 之值；及 (d) E(X^2Y|X=x) 之值。

3. (15%) 投擲一公正的骰子，直到恰好出現兩次大於 4 之點數即停止。令X表總共投擲次數，(a) 寫出X之分佈，並指出其為那一常見分佈，參數為何？另，試求(b) $P(X \geq 5)$，及(c) X之期望值。

4. (10%) 設X與Y獨立，且變異數分別為 Var(X)=2 及 Var(Y)=16。試求
   (a) 變異數 Var(5X-Y)及 (b) 相關係數 $Corr(3X+Y, 2X+3Y)$。

5. (10%) 假設某診所每天掛號之病人數有 *Poisson*(50) 分佈。試利用柴比雪夫不等式，求某日來掛號之病人數介於 40 與 60 人間之一機率下界。

6. (25%) 設$X_1, X_2, ..., X_n$為一組來自常態分佈$N(\mu, \sigma^2)$之隨機樣本。試求(μ, σ^2)之最大概似估計$(\hat{\mu}, \hat{\sigma}^2)$，並分別指出此二估計量之分佈，再以其分別建立$(\mu, \sigma^2)$之 95%信賴區間。（註：分佈之百分位可以符號取代之）

7. (10%) 設$X_1, X_2, ..., X_n$為一組隨機樣本，假定其二階動差存在。試給出一個檢定方法，檢定這組隨機樣本之期望值（平均數）是否等於μ。

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一、選擇題(可複選)

1. (5%)下列函數何者在$x = 0$之極限存在：

(1)$f(x) = \begin{cases} 1, & x < 0 \\ 0, & x \geq 0 \end{cases}$；　(2) $g(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$；　(3) $h(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & x > 0 \\ 0, & x \leq 0 \end{cases}$；

(4) 以上皆不存在。

2. (5%)下列何者非為$f(x)$在$x = a$點連續之必要條件：

(1) $f(a)$有定義；　(2) $f(x)$在$x = a$點之左極限與右極限存在；

(3) $f(x)$在$x = a$點之極限值為$f(a)$；　(4) $f(x)$在$x = a$點上可微。

3. (5%)令 Q 代表有理數之集合。考慮函數$f(x) = \begin{cases} 1, & x \in Q \\ 0, & x \in Q^c \end{cases}$，則$f(x)$在何點連續：

(1) 1；　(2) 2；　(3) 4；　(4)以上皆非。

4. (5%)承上，考慮函數$f(x) = \begin{cases} 1, & x \in Q \\ x, & x \in Q^c \end{cases}$，則$f(x)$在何點連續：

(1) 1；　(2) 2；　(3) 4；　(4)以上皆非。

5. (5%) 假設函數$f(x) = \frac{4x+3}{x-6}$，則下列敘述何者正確：

(1)此函數在$(-\infty, \infty)$連續；(2)此函數之反函數為$f^{-1}(x) = \frac{6x+3}{x+4}$ ；

(3)此函數為奇函數；(4) 以上皆非。

二、是非題

6. (5%)若函數$f(x)$於$x = a$不可微，則$f(x)$於$x = a$必不連續。

7. (5%)若$f(x)$之一階與二階導數於$x = a$之值為零(即$f'(a) = 0$且$f''(a) = 0$)，則$f(a)$必為局部極大或極小值。

8. (5%)考慮一實數列$\{a_n\}$，其中$a_n \to c$。令$f(x)$為一在$\{a_n\}$上均有定義之函數，則$f(a_n) \to f(c)$。

9. (5%) 若$\sum_{n=1}^{\infty} a_n$收斂，則$\lim_{n \to \infty} a_n = 0$。

10. (5%) 若$\sum_{n=1}^{\infty} a_n$發散，則$\lim_{n \to \infty} a_n \neq 0$。

三、問答與計算題

11. 試驗證下列級數收斂或發散。

(1)(5%) $\sum_{n=1}^{\infty} \frac{4^n n! n!}{(2n)!}$；　(2)(5%) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$；　(3)(5%) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3-1}$。

12. 試解下列不定積分式。

(1)(5%)$\int \sin(x)\sin(2x)\sin(4x)dx$；　(2)(5%) $\int \frac{1}{x-\sqrt{1-x^2}}dx$；　(3)(5%) $\int \frac{\sqrt{x}}{1+x^3}dx$。

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13. 考慮函數 $f(x)$ 為一定義在實數域 R 上連續型隨機變數 X 之機率密度函數，則此隨機變數之機率分布函數為 $f(x)$ 之反導函數，定義為 $F(x)$ 且滿足 $F(-\infty) = 0$。若定義一機率函數 $P(a < X \le b) = F(b) - F(a)$。

(1)（10%）試證明連續型隨機變數之任意單點機率函數（$P(X = a), a \in R$）其值為 0。

(2)（10%）試求 $\dfrac{P(X=a)}{P(X=b)}$ 之值（以機率密度函數表示），其中 $a, b \in R$。

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Part I: Choose the best one answer for each of the following questions. (2% each)

1. Refer to Figure 1 below. If these are the only two sellers in the market, then the market quantity supplied at a price of $6 is
   (a). 2 units.
   (b). 10 units.
   (c). 12 units.
   (d). 22 units.

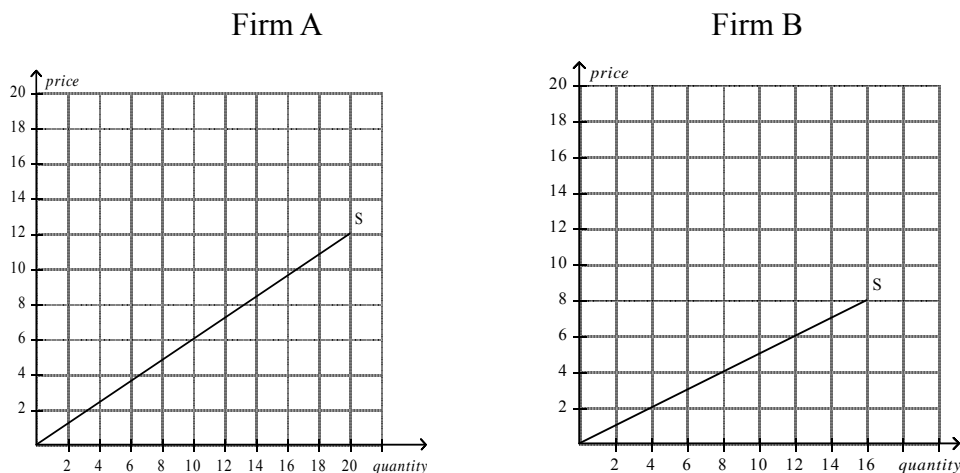   *Figure 1*

   Firm A

   Firm B

   Table 1

	Supply Curve A		Supply Curve B		Supply Curve C	
Price	$1.00	$2.00	$1.00	$3.00	$2.00	$5.00
Quantity Supplied	500	600	600	900	400	700

2. Refer to Table 1 above. Which of the three supply curves represents the least elastic supply?
   (a). supply curve A
   (b). supply curve B
   (c). supply curve C
   (d) There is no difference in the elasticity of the three supply curves.

3. Refer to Table 1 above. Along which of the supply curves does quantity supplied move proportionately more than the price?
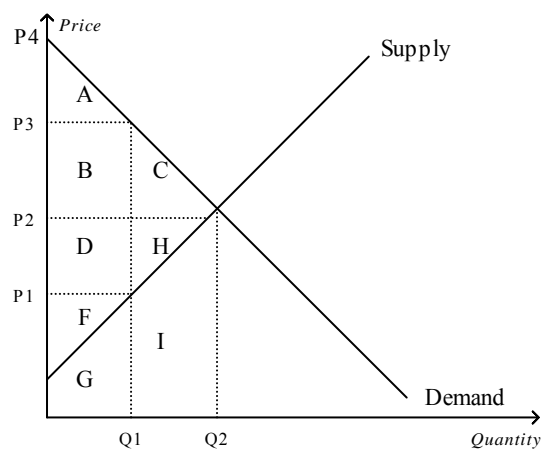   (a). along supply curve B only

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(b). along supply curve B and C

(c). along all three supply curves

(d). Quantity supplied moves proportionately more than the price along none of the three supply curves.

4. Suppose buyers of liquor are required to send $1.00 to the government for every bottle of liquor they buy. Further, suppose this tax causes the effective price received by sellers of liquor to fall by $0.80 per bottle. Which of the following statements is correct?

(a). This tax causes the demand curve for liquor to shift downward by $1.00 at each quantity of liquor.

(b). The price paid by buyers is $0.20 per bottle more than it was before the tax.

(c). Eighty percent of the burden of the tax falls on sellers.

(d). All of the above are correct.

Figure 2

5. Refer to Figure 2. At equilibrium, consumer surplus is represented by the area

(a). A.

(b). A+B+C.

(c). D+H+F.

(d). A+B+C+D+H+F.

6. Refer to Figure 2. If the price were P3, consumer surplus would be represented by the area

(a). A.

(b). A+B+C.

(c). D+H+F.

(d). A+B+C+D+H+F.

7. In the market for widgets, the supply curve is the typical upward-sloping straight line, and the demand curve is the typical downward-sloping straight line. The equilibrium quantity in the market for widgets is 200 per month when there is no tax. Then a tax of $5 per widget is imposed. The price paid by buyers increases by $2 and the after-tax price received by sellers falls by $3. The government is able to raise $750 per month in revenue from the tax. The deadweight loss from the tax is

(a). $250.

(b). $125.

(c). $75.

(d). $50.

8. After a certain nation changed its policy from one that banned international trade in wheat to one that allowed international trade in wheat, the nation began importing wheat. As a result, total surplus in the wheat market increased by $10 million. Which of the following changes could have occurred as well?

(a). The price of wheat in that nation increased with the adoption of the new policy.

(b). The domestic quantity of wheat supplied increased with the adoption of the new policy.

(c). Consumer surplus in the wheat market increased by $7 million and producer surplus in the wheat market increased by $3 million.

(d). Consumer surplus in the wheat market increased by $15 million and producer surplus in the wheat market decreased by $5 million.

9. Some time ago, the nation of Republica opened up its paper market to international trade. Which of the following results of this policy change is consistent with the notion that Republica has a comparative advantage over other countries in producing paper?

(a). The price of paper in Republica decreased as a result of the policy change.

(b). Republica began exporting paper as a result of the policy change.

(c). The domestic demand curve for paper shifted to the right as a result of the policy change.

(d). The domestic quantity of paper demanded increased as a result of the policy change.

10. Which of the following statements is true of the tax on gasoline?

(a). The cost of collecting a gasoline tax outweighs the revenues raised by the tax.
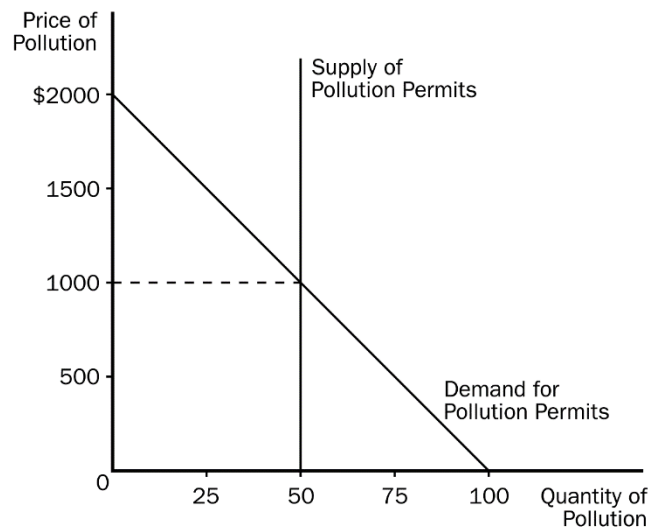
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(b). It is preferred to tolls as the best solution to road congestion.

(c). It discourages driving on noncongested roads, even though there is no congestion externality for these roads.

(d). Both b and c are correct.

Figure 3

11. Refer to Figure 3. This graph shows the market for pollution when permits are issued to firms and traded in the marketplace. The equilibrium price of pollution is
    (a). $50
    (b). $500
    (c). $1,000
    (d). $2,000

12. Refer to Figure 3. This graph shows the market for pollution when permits are issued to firms and traded in the marketplace. The equilibrium number of permits is
    (a). 50
    (b). 100
    (c). 1,000
    (d). 2,000

13. Which of the following tax systems could not be structured to satisfy conditions of vertical equity?

(a). A proportional tax

(b). A regressive tax

(c). A progressive tax

(d). A lump-sum tax

14. The fundamental reason that marginal cost eventually rises as output increases is because of

(a). economies of scale.

(b). diseconomies of scale.

(c). diminishing marginal product.

(d). rising average fixed cost.

15. Consider the following information about bread production at Beth's Bakery:

Worker	Marginal Product
1	5
2	7
3	10
4	11
5	8
6	6
7	4

Beth pays all her workers the same wage, and labor is her only variable cost. From this information we can conclude that Beth's marginal cost

(a). declines as output increases from 0 to 33, but increases after that.

(b). declines as output increases from 0 to 11, but increases after that.

(c). increases as output increases from 0 to 11, but declines after that.

(d). is constant.

16. A firm will shut down in the short run if, for all positive levels of output,

(a). its loss exceeds its fixed costs.

(b). its total revenue is less than its variable costs.

(c). the price of its product is less than its average variable cost.

(d). All of the above are correct.

17. Many movie theaters allow discount tickets to be sold to senior citizens because

(a). senior-citizen laws mandate such discounts.

(b). goodwill efforts show community respect and win loyal patrons.

(c). the theaters are profit maximizers.

(d). senior citizens lobby city councils for lower prices.

18. Which of the following markets impose deadweight losses on society?

(i)　perfect competition

(ii)　monopolistic competition

(iii)　monopoly

(a). (i) and (ii) only

(b). (ii) and (iii) only

(c). (i) and (iii) only

(d). (i) only

Table 2. Brian and Matt own the only two bicycle repair shops in town. Each must choose between a low price for repair work and a high price. The annual economic profit from each strategy is indicated in the table. The profits are shown as (Matt, Brian) in each cell.

		Brian	
		Low Price	*High Price*
Matt	*Low Price*	(1500, 1500)	(5000, 200)
	High Price	(200, 3000)	(4000, 4000)

19. Refer to Table 2. Which of the following statements is correct?

(a). Matt's dominant strategy is to charge a low price.

(b). Brian's dominant strategy is to charge a high price.

(c). The dominant strategy for both Brian and Matt is to charge a low price.

(d). Matt's dominant strategy is to charge a high price.

20. Refer to Table 2. Which of the following statements is correct if Brian and Matt will play this game only once?

(a). The Nash equilibrium is the high price.

(b). A Nash equilibrium cannot be established unless Brian and Matt collude.

(c). A Nash equilibrium cannot be established without the players repeating the game.

(d). The Nash equilibrium price is the low price.

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21. Suppose that a competitive firm hires labor up to the point at which the value of the marginal product equals the wage.   If the firm pays a wage of $700 per week and the marginal product of labor equals 100 units per week, then the marginal cost of producing an additional unit of output is
    (a). $7
    (b). $70
    (c). $700
    (d). We do not have enough information to answer this question.

22. Wage differences that can't be explained by the number of years of training could result from
    (a). demand but not supply.
    (b). supply but not demand.
    (c). compensating differentials.
    (d). the marginal product of capital.

23. The poverty line in the country of Inequalia is $10,000. The distribution of income for Inequalia is as follows:

Number of Families	Income
200	less than $5,000
300	between $5,000 and $10,000
500	between $10,000 and $15,000
700	between $15,000 and $20,000
100	over $20,000

The poverty rate in Inequalia is
    (a). 11.1 percent.
    (b). 16.7 percent.
    (c). 27.8 percent.
    (d). 55.5 percent.

24. Karen, Tara, and Chelsea each buy ice cream and paperback novels to enjoy on hot summer days.   Ice cream costs $5 per gallon, and paperback novels cost $8 each.   Karen has a budget of $80, Tara has a budget of $60, and Chelsea has a budget of $40 to spend on ice cream and paperback novels.   Which of the following statements is correct?
    (a). Each woman faces the same budget constraint.
    (b). The slope of the budget constraint is the same for each woman.

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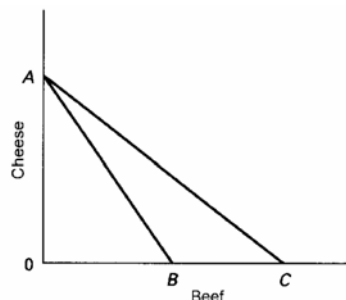系所：統計學研究所(風險管理組)
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(c). The area underneath the budget constraint is larger for Chelsea than for Karen.

(d). All of the above are correct.

Table 3. Three longtime friends-Fred, Barney, and Wilma-are deciding how they will spend their Sunday afternoon. They all agree that they should do one of three things: go to a movie, go to the beach, or go to a museum. They also agree that they will have two pairwise votes to determine how to spend their evening, with the majority determining the outcome on each vote. The first, second, and third choices for each person are as indicated in the table below.

	Fred	**Barney**	**Wilma**
First choice	Museum	Beach	Movie
Second choice	Beach	Movie	Museum
Third choice	Movie	Museum	Beach

25. Refer to Table 3. If (1) the first vote pits "museum" against "movie," and (2) the second vote pits "beach" against the winner of the first vote, then the outcome is as follows:

    (a). "Museum" wins the first vote and "museum" wins the second vote, so they go to a museum.

    (b). "Museum" wins the first vote and "beach" wins the second vote, so they go to the beach.

    (c). "Movie" wins the first vote and "movie" wins the second vote, so they go to a movie.

    (d). "Movie" wins the first vote and "beach" wins the second vote, so they go to the beach.

26. Refer to Table 3. Which of the following statements is correct?

    (a). In a pairwise election, "movie" beats "beach."

    (b). In a pairwise election, "beach" beats "museum."

    (c). In a pairwise election, "museum" beats "movie."

    (d). None of the above is correct.

Figure 4. Use the following to answer questions 27-28:

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27. Refer to the above diagram in which line AB is the United States production possibility curve and AC is its trading possibilities curve. We can conclude that the United States:
    (a). has chosen to specialize in the production of cheese.
    (b). has chosen to specialize in the production of beef.
    (c). has decided to trade beef for cheese.
    (d). is relatively more efficient than its trading partners in producing both cheese and beef.

28. Refer to the above diagram in which line AB is the United States production possibility curve and AC is its trading possibilities curve. The international exchange ratio between beef and cheese (terms of trade):
    (a). is the absolute value of slope of line AB.
    (b). is the absolute value of slope of line AC.
    (c). could lie anywhere between the absolute value of the slopes of lines AB and AC.
    (d). cannot be determined on the basis of this information.

29. Which attitude or custom is the most conducive to long-term economic growth?
    (a). focus on group contentment rather than individual achievement
    (b). the belief there is little or no correlation between an individual's economic actions and her or his economic fortunes.
    (c). direct connections between individual efforts (including educational efforts) and economic rewards
    (d). use of the majority of resources for religious structures and ceremonies

30. The fact that international specialization and trade based on comparative advantage can increase world output is demonstrated by the reality that:
    (a). the production possibilities curve of any two nations are identical.
    (b). a nation's production possibilities and trading possibilities lines coincide.
    (c). a nation's trading possibilities line lies to the right of its production possibilities line.
    (d). a nation's production possibilities line lies to the right of its trading possibilities line.

Part II: Answer the following problems.

1. Suppose that Intel has a monopoly in the market for computer chips. In order to produce X computer chips, it costs Intel $C(X) = 2X^2$.
    (a) Find the marginal cost of producing a computer chip for Intel.   (2%)
    (b) The demand for computer chips is $X_D = 12 - 0.25P$. Find the level of output that maximizes

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Intel's profits. What price is Intel charging?　　(6%)

(c) What level of output would maximize total surplus in the computer chip market?　　(4%)

(d) Suppose the government knew the demand and production functions. Find a price regulation the government could impose that would induce Intel to maximize total surplus, i.e., produce the efficient quantity from part (c).　　(4%)

(e) If the government subsidized Intel s for every unit of computer chips produced, what quantity would Intel choose as a function of s? Find the choice of subsidy that maximizes total surplus, i.e., induces Intel to produce the efficient quantity from part (c).　　(6%)

2.　Firm 1 and firm 2 are the only producers of spring water in the market. The market demand for spring water is given by $P = 70 - Q_1 - Q_2$. Firm 1 and firm 2 compete by choosing quantities Q_1 and Q_2 respectively. Each firm has a marginal cost of 10 and no fixed cost.

(a) Find out firm 1's and firm 2's reaction functions.　　(6%)

(b) Suppose the two firms choose quantities simultaneously. What are the equilibrium price, quantities, and profits of the two firms in this market?　　(6%)

(c) Suppose only firm 1 has a chance to bribe the government and get the right to choose the quantity first, what is the maximum amount of money that firm 1 is willing to pay? If firm 1 gets to move first, what are the equilibrium quantities and profits of firm 1 and firm 2?　　(6%)