招 生 學 年 度	106	招 生 類 別	碩士班
系 所 班 別	應用數學系碩士班、應用數學系 統計碩士班		
科 目 名 稱	基礎數學		
注 意 事 項	含微積分及線性代數		

(1) Evaluate the followings

  (a) (5%) $\frac{d\sqrt{e^{\sin x}}}{dx}$

  (b) (5%) $\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}$

(2) (10%) Evaluate $\int \frac{1 + 3\sqrt{x} + 2x}{1 + \sqrt{x}} dx$

(3) (10%) Let $\mathbf{u} = (1, 2)$, $\mathbf{v} = (1, -1)$ and A be a linear map $\mathbb{R}^2 \to \mathbb{R}^2$, such that $A\mathbf{u} = (5, 2)$ and $A\mathbf{v} = (3, 1)$. Suppose $A\mathbf{w} = (3, 2)$. Find $\mathbf{w}$.

(4) (10%) Compute $\det \begin{pmatrix} 2 & 3 & 5 & 7 \\ 3 & 5 & 7 & 11 \\ 5 & 7 & 11 & 13 \\ 7 & 11 & 13 & 17 \end{pmatrix}$.

(5) Let W be the linear span of $\{(1, 1, 1, 1, 1), (3, 2, 1, 0, 4), (-1, 5, 5, 9, 2)\}$ in $\mathbb{R}^5$.

  (a) (10%) Find an orthonormal basis of W.

  (b) (10%) Find the distance between $p = (1, 2, 3, 4, 5)$ to W (the distance between p and the projection point of p on W).

(6) (10%) Construct a 2 by 2 real matrix M, so that M has eigenvalues $1, 2$ and eigenvectors $(1, 1)$, $(1, -1)$ respectively.

(7) (10%) Find the slope of the line tangent to the curve $y^2 + xy + x^3 = 3$ at $(1, 1)$.

(8) (10%) Find the maximum and minimum value of $e^x \cos x$ in $[-\pi, \pi]$.

(9) Determining the convergence or divergence of following series. Show the test you used for each of the series.

  (a) (2%) $\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}}$

  (b) (2%) $\sum_{n=1}^{\infty} \frac{100^n}{n!}$

  (c) (2%) $\sum_{n=0}^{\infty} \frac{n}{10n^2 + 5n + 7}$

  (d) (2%) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

  (e) (2%) $\sum_{n=1}^{\infty} (-1)^n$