

招生學年度	106	招生類別	碩士班
系所班別	運籌管理研究所碩士班、財務金融學系碩士班（金融大數據組）		
科目名稱	微積分		
注意事項	本考科可使用掌上型計算機		

1. 試應用夾擊定理求解  $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x}$  (10%)
2. Find the following integrals: (a)  $\int \frac{1}{\sqrt{3x^2 - 1}} dx$  (b)  $\int x^2 \ln x dx$  (10%)
3. If  $f(x) = x^5$ ,  $g(x) = -7x$ , find  $(f \circ g)'(x)$  and  $(g \circ f)'(x)$  (10%)
4. 試證明  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$  (15%)
5. Let  $f(x, y) = \frac{x^3 - xy^2}{x^2 + y^2}$ , if  $(x, y) \neq (0, 0)$ , and  $f(0, 0) = 0$ .  
Find (a)  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  (b)  $f_y(0, 0)$  (15%)
6. Let  $f(x, y) = \frac{\sin(x - y)}{\cos(x + y)}$ , Find  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  (10%)
7. Evaluate the following integrals: (a)  $\int_0^{\frac{\pi}{4}} \frac{3 \cos \theta}{\sqrt{2 - \sin^2 \theta}} d\theta$  (10%)
8. Check the convergence or divergence of the following series.  

$$\sum_{n=1}^{\infty} \frac{3}{2 + e^n}$$
 and give a proof. (20%)