一、離散數學 (共 50 分)

1. (a) How many arrangements are there for all the letters in “ALALABAMA” ?　(5%)
   (b) How many of arrangements in part (a) have no adjacent A?　(5%)

2. Let $p(x)$ be the following open statement.

   $p(x, y): x + y = 20$

   Determine the truth or falsity of the following statements, where x and y are integers.

   (a) $\exists y \forall x\, p(x, y)$　(4%)

   (b) $\forall x \exists y\, p(x, y)$　(4%)

3. Dertermine $|A \cup B \cup C|$ when $|A|=30$, $|B|=300$, and $|C| = 3000$, if
   (a) $A \subseteq B \subseteq C$　(4%)
   (b) $A \cap B = A \cap C = B \cap C = \varnothing$　(4%)
   (c) $|A \cap B| = |A \cap C| = |B \cap C| = 3$, and $|A \cap B \cap C| = 1$　(4%)

4. For positive integer $n > 4$, prove that $2^n > n^2$ by mathematical induction.　(10%)

5. Let $A = \{1, 2, 3, 4, 5\}$. Define the relation $\mathcal{R}$ on A by $x\, \mathcal{R}\, y$, if $x + y = 6$

   (a) List the set of $\mathcal{R}$.　(5%)

   (b) Does the relation $\mathcal{R}$ satisfy the properties of reflexive, symmetric, anti-symmetric, and transitive?　(5%)

二、線性代數 (共 50 分)

1. State (with a brief explanation) whether the following statements are true or false. No grade is given if there is no explanation provided for your answer.　(20%)

   (a) The set {(1, 0), (2, 0)} is a basis for R^2.

   (b) The vectors (0, 1, 0), (2, 0, 0), (0, 0, 3) span R^3.

   (c) The set $U = \{(a, b, c) \mid a + b + c = 1,\ a, b, c \in R\}$ is a subspace of R^3.

   (d) Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 4 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$, and A is a singular matrix.

   (e) Let $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 5 & 1 & 2 & 6 \\ 0 & 1 & 0 & 4 \\ 0 & 2 & 0 & 0 \end{bmatrix}$, and A is invertible.

2. Prove that the following transformation $T: R^3 \rightarrow R^2$ is not linear. (10%)

$$T(x, y, z) = (x + y, z + 1)$$

3. Solve the following system using LU decomposition, where L is a lower triangular matrix and U is an upper triangular matrix. Show L, U and solutions for x_1, x_2, x_3. (10%)

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 1 \\ 2x_1 - 5x_2 + 12x_3 &= 3 \\ 2x_2 - 10x_3 &= 0 \end{aligned}$$

4. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$. (10%)