

1. (10 points) Use Cramer's rule to solve the following system

$$4x_2 + 5x_3 = 3$$

$$2x_1 + 3x_2 = 8$$

$$6x_1 + 7x_3 = -1$$

2. $A = \begin{bmatrix} 1 & 7 & 5 \\ 3 & -1 & 1 \\ 2 & -2 & 0 \end{bmatrix}$

- (a) (5 points) Find the inverse matrix of A .
(b) (10 points) Compute the LU factorization of the matrix A

3. $B = \begin{bmatrix} 3 & -2 & 1 \\ 1 & 0 & 7 \\ 0 & 0 & 2 \end{bmatrix}$

- (a) (10 points) Find the eigenvalues of B and corresponding eigenvectors.

(b) (5 points) Determine $B^{50} * \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$

- (c) (10 points) Find the geometric and algebraic multiplicities of each eigenvalue.

4. (10 points) List four basic operations with the calling prototype (including name of the operation; Parameters if any; return value if any) for the abstract data type **Binary Search Trees**.

5. (10 points) Draw a figure with a brief description (around 50 English words) to define what abstract data type (ADT) is.

6. (10 points) *Arrays* and *Linked List* can be used to store data, but both of them have some advantages and disadvantages over each other. Compare the advantages and disadvantages of these two types of implementations.
7. (10 points) List the time complexity of the following 5 different sorting algorithms in the following table.

	Sort algorithm	Time Complexity
1	Bubble Sort	
2	Quick Sort	
3	Selection Sort	
4	Heap Sort	
5	Insertion Sort	

8. (10 points) Draw figures for the heap after the operations (insert node "67") and (delete node "78").

