國立臺灣師範大學105學年度碩士班招生考試試題

科目：**機率與統計**　　適用系所：**數學系**

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。
3.答案必須有計算過程，若沒有計算過程，會斟酌扣分。

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1. (20 points)Suppose (X, Y) is uniformly distributed over the region $\{(x, y) : 0 < |x| + |y| < 1\}$.

   (a) Find the joint probability density function of (X, Y).

   (b) Find the marginal density functions $f_X(x)$ and $f_Y(y)$.

   (c) Are X and Y independent?

2. (20 points)Consider Jane and Chuck roll n, $n = 1, 2, \ldots$, independent fair six-sided dice respectively. Let X_n and Y_n be the sum of n dice rolled by Jane and Chuck respectively. Find the following probabilities.

   (a) $P(X_1 < Y_1)$.

   (b) $P(X_2 < Y_2)$.

   (c) $\lim_{n\to\infty} P(X_n < Y_n)$.

3. (20 points)Suppose X follows a normal distribution with mean 0 and variance θ^2. $\theta > 0$

   (a) Is X sufficient and complete for θ?

   (b) Find the minimum variance unbiased estimator (MVUE) of θ, if it exists.

4. (20 points)Let X_1, X_2 be a random sample having p.d.f. $f(x;\theta) = \begin{cases} \frac{1}{\theta}e^{-\frac{x}{\theta}}, & x > 0 \\ 0, & \text{otherwise,} \end{cases}$ with the parameter space $\Omega = \{\theta|\theta = 1, 2\}$. To test $H_0 : \theta = 2$ vs $H_a : \theta = 1$, we reject H_0 iff $\frac{f(x_1;2)f(x_2;2)}{f(x_1;1)f(x_2;1)} \leq \frac{1}{2}$.

   (a) Find the significance level of the test.

   (b) Find the power of the test when H_0 is false.

5. (20 points)Let X_i, $i = 1,...,m$, be independent random variables following the distribution of Binomial(n_i, p_i).

   (a) Derive a likelihood ratio test statistic for the hypothesis $H_0 : p_1 = p_2 = ... = p_m$ against the alternative that $p_i \neq p_j$, for some $1 \leq i < j \leq m$.

   (b) What is the asymptotic distribution of the test statistic in part (a)?