考試科目	統計學B 41221	所別	金融學系財務工程組	考試時間	2月 18日(六)第二節

1. Let X_t be a return and follow a normally distributed random variable with mean $\left(r-\frac{1}{2}\sigma^2\right)t$ and variance $\sigma^2 t$, i.e. $X_t \sim N\left((r-\frac{1}{2}\sigma^2)t, \sigma^2 t\right)$. If $S_t = S_0 e^{X_t}$ where S_t denotes the stock price at time t, then the stock price S_t is said to be lognormally distributed random variable.

(1).(10%) Please show that $e^{-rt}S_t$ is martingale, and find the price of the futures at time 0. (i.e., what is $E(S_t)$)

(2).(10%) Find the probability $P(S_t > K)$ for a positive real number K and a given cumulative standard normal distribution $N(\bullet)$ where $N(d)=\int_{-\infty}^{d}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{y^2}{2}\right\}dy$. (i.e., the probability which the stock price exceeds K at time t).

(3).(15%) Compute the price of the call option at no-arbitrage opportunity, $E\left(e^{-rt}\left(S_t - K\right) 1_{\{S_t > K\}}\right)$, where $1_{\{S_t > K\}}$ denotes the indicator function. (i.e, the expectation of the profit $\left(S_t - K\right)$ when the stock price exceeds K at time t).

(4). (5%) Please explain what is the no-arbitrage opportunity or condition for the problem (3).

(5).(10%) If the answer of the problem (3) is the pricing formula of the call option with the underlying stock, please find five parameters for the pricing formula, which parameter have to estimate, and how to estimate the parameter.

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2. A stochastic process $\{X(t), t \geq 0\}$ is said to be a compound Poisson process if it can be represented as

$$X(t) = \sum_{i=1}^{N(t)} Y_i, \qquad t \geq 0$$

where $\{N(t), t \geq 0\}$ is a Poisson process with a arrival rate λt, and $\{Y_i, i \geq 0\}$ is a family of independent and identically distributed random variables from an exponential distribution with the parameter β, where the density function is $f(y) = \beta \exp(-\beta y)$. Assume that $\{Y_i, i \geq 0\}$ are also independent of $\{N(t), t \geq 0\}$.

(1).(10%)Please find the mean and variance of $X(t)$.

(2).(10%) Please find the mean and variance of $\exp\{X(t)\}$. (i.e., $A = E(\exp\{X(t)\}) = \exp(kt)$, $B = Var(\exp\{X(t)\})$)

(3).(15%) Please show $\exp\{X(t) - kt\}$ to be martingale and find k.

(4).(15%) Please find the estimators λ, μ and σ^2 when you have the data of 2 samples with the arrival numbers $(n_1, n_2, \ldots, n_t)$ and the values of $\{(y_1, y_2, \ldots, y_{n_1}), (y_1, y_2, \ldots, y_{n_2}), \ldots, (y_1, y_2, \ldots, y_{n_t})\}$ based on maximum likelihood or the method of moment estimation?

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。