國立臺北大學106學年度碩士班一般入學考試試題

系（所）組別： 統計學系
科　　　目： 基礎數學

□可 ☑不可使用計算機

一、Calculus

1. Let $f(x,y)=\begin{cases}\dfrac{x^3-y^3}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0 & ,(x,y)=(0,0)\end{cases}$.

   (a) (6%) Find $f_x(x,y)$ for $(x,y)\neq(0,0)$.

   (b) (7%) Find $f_x(0,0)$.

   (c) (7%) Prove or disprove that $f_x(x,y)$ is continuous at $(0,0)$.

2. Let $f(x)=\ln x$, $x>0$.

   (a) (8%) Find the Taylor's series of $f(x)$ centered at 1.

   (b) (7%) Find the interval of convergence of the power series in (a).

3. (a) (7%) Evaluate $\int_0^1 x\tan^{-1}x\,dx$.

   (b) (8%) Evaluate $\int_0^1\int_0^{2-2x} e^{\frac{y}{2x+y}}\,dy\,dx$ by setting $u=2x+y$, $v=y$.

二、所有題目請敘述計算過程，無計算過程不給分

1. Let $A=\begin{pmatrix}-3 & 0 & -1\\ 0 & -2 & 0\\ -1 & 0 & -3\end{pmatrix}$.

   (a) Find the inverse of A. (5%)

   (b) Find the eigenvalues and the corresponding eigenspaces of A. (5%)

   (c) Please orthogonally diagonalize the matrix A. (You need to find out an orthogonal matrix P and a diagonal matrix D such that $P^TAP=D$.) (10%)

   (d) If A is the matrix representing the linear transformation $L: P_2\to P_2$ with respect to the basis $\{t^2+1, t, t-1\}$, please compute $L(t^2+t+1)$. (10%)

2. A symmetric $n\times n$ matrix A is called positive definite if $\vec{x}^TA\vec{x}>0$ for every nonzero vector $\vec{x}$ in R^n. Suppose that $B=P^TP$ for an $n\times n$ nonsingular matrix P. Please show that B is positive definite. (10%)

3. Let $S=\{\vec{u}_1,\vec{u}_2,\dots,\vec{u}_k\}$ be an orthogonal set of nonzero vectors in R^n. Please show that S is linear independent. (10%)

試題隨卷繳交