

國立臺北大學 106 學年度碩士班一般入學考試試題

系（所）組別：金融與合作經營學系

科目：統計學

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☒可 ☐不可使用計算機

一、(10%) A random variable X , $E(X)=15$, $P(X>20)=0.5$, $P(X<10)=0.2$. Please calculate the lower bound of $\text{Var}(X)$.

二、(26%) The effective life (unit: hours) of an electronic component follows exponential distribution with density $f(x) = e^{-x}$, $x > 0$.

(a) (14%) A system is composed by 2 independent such components which are serially connected (it means if one component broken, the system will not work). Derive the density function of the system's life. What is the probability of the system's life longer than 2 hours?

(b) (12%) Another system is also composed by 2 independent such components but are parallel connected (it will still work until both components broken), derive the density function of this system's life.

三、(14%) $X \sim N(0,1)$. Please derive the moment generating function of X , and calculate the fourth moment $E(X^4)$.

四、(15%) X_1, \dots, X_n are independent variables and all follow $N(\mu, \sigma^2)$.

(a) (5%) Please show $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ is a unbiased estimator of σ^2 , where $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$.

(b) (10%) Please show $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$ is a consistent estimator of σ^2 .

五、(10%) Let X_1, \dots, X_n denote a random sample from the binomial distribution $b(1, p)$. $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ is an unbiased estimator of p and $\text{Var}(\bar{X}) = p(1-p)/n$

(a) (5%) Please find the Rao-Cramér lower bound for \bar{X} .

(b) (5%) What is the efficiency of \bar{X} as an estimator of p .

六、(15%) Randomly select 20 cars of each of two comparable major-brand models. All cars are submitted to accelerated life testing. Namely, these cars are driven many miles over very poor roads in a short time. Their failure times (unit: week) are summarized as the following table. Use a chi-square test with $\alpha=0.05$ to test the equality of the distributions of accelerated life testing for the two brands.

Brand \ Weeks	< 23.5	23.5 ~ 28.5	28.5 ~ 34.5	≥ 34.5	Total
Brand U	2	4	4	10	20
Brand V	8	6	6	0	20

($\chi_{0.05}^2(2)=5.991$; $\chi_{0.05}^2(3)=7.815$; $\chi_{0.05}^2(4)=9.488$; $\chi_{0.05}^2(8)=15.51$)

七、(10%) Suppose that $P(A) = 0.7$, $P(B) = 0.5$, and $P((A \cup B)^C) = 0.1$, where the superscript C denotes the complement with respect to the universal set.

(a) (7%) Find $P(A \cap B)$.

(b) (3%) Find $P(A|B)$.