考試科目	統計學B 41221	所別	金融學系	考試時間	2 月27 日(六)第二節

1. Let X and Y be independent Poisson random variables with parameters λ_X and λ_Y, respectively.

   (a) Show that $P(X+Y=n)=\sum_{i=0}^{n}P(X=i)P(Y=n-i)$. (5%)

   (b) Use part (a) to prove that $X+Y$ is a Poisson random variable with a parameter $\lambda_X+\lambda_Y$. (5%)

   (c) Please calculate the conditional probability and conditional expectation of X given that $X+Y=n$. (10%)

2. A stochastic process $\{X(t), t\geq 0\}$ is said to be a compound Poisson process if it can be represented as

$$X(t)=\sum_{i=1}^{N(t)}Y_i, \qquad t\geq 0$$

where $\{N(t), t\geq 0\}$ is a Poisson process with a arrival rate λ, and $\{Y_i, i\geq 0\}$ is a family of independent and identically distributed random variables from normal density with mean μ and variance σ^2 which are also independent of $\{N(t), t\geq 0\}$.

   (a). Please find the mean of $\exp\{X(T)\}$ at the time T. (5%)

   (b). Please show $\exp\{X(T)-\lambda(k-1)T\}$ to be martingale, where $k=\exp\{\mu+0.5\sigma^2\}$. (5%)

   (c). Please find the estimators λ, μ and σ^2 when you have the data of 2 samples with the arrival numbers $(n_1, n_2, \ldots, n_T)$ and the values $\{(y_1, y_2, \ldots, y_{n_1}), (y_1, y_2, \ldots, y_{n_2}), \ldots, (y_1, y_2, \ldots, y_{n_T})\}$? (10%)

   (d). If the arrival numbers are missing or unobservable, how to estimate λ, μ and σ^2? (10%)

備註	試題隨卷繳交

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3. Let the dynamics of the stock price be $S(T) = S(0)\exp\left\{(r - \frac{1}{2}\sigma^2)T + \sigma B(T)\right\}$ under the risk neutral measure at time T, where $S(0)$ denotes the stock price at time 0, r is the riskless rate, σ is the volatility of the log stock price, and $\{B(t), t \geq 0\}$ is a Brownian motion process with $B(0) = 0$. $B(t)$ is normal distribution with mean 0 and variance t at time t, where its density function is given by

$$f_t(b) = \frac{1}{\sqrt{2\pi t}} e^{-b^2/2t},$$

and the process $B(t)$ has stationary and independent increments, where $B(t_1)$, $B(t_2) - B(t_1)$, ..., $B(t_n) - B(t_{n-1})$ for $t_1 < ... < t_n$ are independent and $B(t_k) - B(t_{k-1})$ is normal with mean 0 and variance $t_k - t_{k-1}$.

(a). Please describe what is the risk-neutral probability measure and the physical (real) probability measure? (10%)

(b). Please find the mean and variance of $S(T)$ under the risk-neutral probability measure. (10%)

(c). If the underlying asset of the futures is the stock, what is the theoretical value of the futures with the maturity T at time 0 under the risk-neutral probability measure? (10%)

(d). If the underlying asset of the option is the stock, what is the theoretical value of the stock option with strike price K and maturity T at time 0 under the risk-neutral probability measure ? (Hint: To derive Black-Scholes Option Pricing Formula.) (10%)

(e). Please find the estimators of μ and σ by the maximum likelihood estimation (MLE) at the physical (real) probability measure based on the stock prices data for n days. (10%)

備 註	試 題 隨 卷 繳 交