國立政治大學 101 學年度研究所 碩士班招生考試試題

考試科目	數理統計學	所別	4141 統計學系	考試時間	2月 26 日(日) 第三節

1. The number of typing errors per page Y of a research assistant is known to have a Poisson distribution with mean λ. However, λ varies from page to page and has an exponential distribution with mean five.
   a. Find the expected number of typing errors per page of the research assistant. (5 pts)
   b. Find the probability density function for Y. (5 pts)

2. In the production of a certain type of copper, two types of copper powder (A and B) are mixed together and heated for a certain length of time. For a fixed volume of heated copper, the producer measures Y_1, the proportion of the volume due to solid copper, and Y_2, the proportion of the solid mass due to Type A powder. Assume that the probability density functions for Y_1 and Y_2 are

$$f_1(y_1)=\begin{cases}6y_1(1-y_1), & 0<y_1<1\\ 0, & elsewhere\end{cases},\quad f_2(y_2)=\begin{cases}3y_2^2, & 0<y_2<1\\ 0, & elsewhere\end{cases}.$$

   Y_1Y_2 is then the proportion of the sample volume due to Type A powder. Assume that Y_1 and Y_2 are independent.
   a. Find the probability density function for Y_1Y_2. (10 pts)
   b. Find $E(Y_1Y_2)$. (10 pts)

3. Let $Y_1, \ldots, Y_n$ be a random sample from a population with probability density function

$$f(y\mid\theta)=\begin{cases}cy^{c-1}/\theta^c, & 0\le y\le\theta,\\ 0, & elsewhere\end{cases}$$

   where $c>0$ is a known constant.
   a. Find the MLE (Maximum Likelihood Estimator) of θ. (5 pts)
   b. Find a minimal sufficient statistic for θ. (5 pts)
   c. Find a MVUE (Minimum Variance Unbiased Estimator) for θ. (10 pts)

備註	試題隨卷繳交

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4. Let $X_1, \cdots, X_n$ be a random samples from an exponential distribution with mean $1/\lambda$, $f(x) = \lambda\exp(-\lambda x)$ for $x > 0$, and assume that the prior density of λ also is exponential with mean $1/\theta$ where θ is known.

(a) Find the posterior distribution of $\lambda|x_1, \cdots, x_n$. (7%)

(b) Using squared error loss, find the Bayes estimator of λ. (7%)

(c) Using absolute error loss, find the Bayes estimator of $1/\lambda$. (6%)

5. Consider a random sample of size n from a distribution with $f(x) = (\frac{2x}{\theta})e^{-x^2/\theta}$ if $x > 0$, zero otherwise. Find a uniformly most powerful test (UMPT) of size α of $H_0: \theta \leq \theta_0$ vs. $H_1: \theta > \theta_0$. (10%)

6. Assume $Y_1, \cdots, Y_n$ are uncorrelated random variables with $E(Y_i) = \beta_0 + \beta_1 x_i$ and $Var(Y_i) = \sigma^2$, and that $\hat{\beta}_0$ and $\hat{\beta}_1$ are the Least Squares estimators of β_0 and β_1. Calculate each of the following:

(a) $Cov(\hat{\beta}_1, \bar{Y})$. (5%)

(b) $E(\hat{\beta}_0^2)$. (5%)

(c) $E(\hat{\beta}_1^2)$. (5%)

(d) Show that $\hat{\sigma}^2 = \sum_{i=1}^{n}\left(Y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i\right)^2/(n-2)$ is an unbiased estimator of σ^2. (5%)

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