考 試 科 目	統計學 A	所 別	4121 金融學系金融管理組	考 試 時 間	2 月 26 日(日)第 2 節

1. (10%) You are running a maximum likelihood program to estimate a dynamic stochastic model. Let X denote the set of endogenous variables and Θ be the set of parameters. However, the program always fails to give the solution. To find out the problem, you obtain the following summary of log-likelihood and of one of the parameters ψ, $\psi \in \Theta$:

ψ	-2.55	-0.87	0.63	2.76	5.25
$\ln f(X;\Theta)$	-120.2525	-120.2528	-120.2521	-120.2522	-120.2526

Suppose that you can exclude any possibilities of technical (hardware, software and programming) issues. Explain why maximum likelihood fails in this case and propose a feasible solution to this problem.

2. You are estimating a linear stationary time-series model $y_t = \beta_0 + \beta_1 x_t + u_t$, $t = 1, 2, ..., T$. However, only x_t^* is observable and $x_t^* = x_t + w_t$. Thus the best you can do is to estimate $y_t = \beta_0 + \beta_1 x_t^* + \epsilon_t$. Assume that (1) u_t has mean zero, variance σ_u^2 and is uncorrelated with x_t and w_t; (2) w_t has mean zero, variance σ_w^2 and is uncorrelated with x_t; (3) each variable is correlated over time but not correlated with past values of other variables.

   (a) (10%) Explain why the OLS estimator $\widehat{\beta}_{1,OLS}$ is inconsistent by checking $Cov(x_t^*, \epsilon_t)$.

   (b) (10%) Consider another estimator $\widehat{\beta}_1$ where

$$\widehat{\beta}_1 = \frac{\sum_{t=1}^{T}(y_t - \overline{y})(x_{t-1}^* - \overline{x}^*)}{\sum_{t=1}^{T}(x_t^* - \overline{x}^*)(x_{t-1}^* - \overline{x}^*)}.$$

   Show that $\operatorname*{plim}_{n\to\infty} \widehat{\beta}_1 = \beta_1$.

3. A river flood warning system records the water mark everyday. Suppose that the low-water mark is set at 1 and the high-water mark Y has the following distribution function:

$$F(y) = 1 - \frac{1}{y^2}, 1 \le y < \infty.$$

   (a) (10%) Find the mean and standard deviation of the high-water mark.

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(b) **(10%)** Suppose that the low-water mark is reset at 0 and the unit of measurement is changed to half of the previous one. Let Z denote the new high-water mark. Find the pdf of Z, i.e. $f(z)$.

4. Let R_{t+1} be the gross return of holding a risky asset from period t to $t+1$ and R^f_{t+1} be the risk-free rate. The excess return is defined as $RP_{t+1} = R_{t+1} - R^f_{t+1}$. These assets can be priced as follows:

$$E_t(M_{t+1}R_{t+1}) = 1 = E_t(M_{t+1}R^f_{t+1}),$$

where M_{t+1} is the stochastic discount factor (SDF) and E_t denotes the expectation operator conditional on time t information.

(a) **(10%)** Show that the Sharpe ratio can be expressed as

$$\frac{E_t(RP_{t+1})}{\sigma_t(RP_{t+1})} = -\rho_t(M_{t+1}, R_{t+1})\frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})},$$

where σ_t denotes the conditional standard deviation operator and ρ_t is the conditional correlation operator.

(b) **(10%)** Let $M_{t+1} = \beta(C_{t+1}/C_t)^{-\gamma}$ where C_t denotes the aggregate consumption, β and γ are constants. Define $\Delta c_{t+1} = c_{t+1} - c_t = \ln(C_{t+1}/C_t)$, $m_{t+1} = \ln M_{t+1}$ and $r^f_{t+1} = \ln R^f_{t+1}$. The logarithm of consumption growth follows a random-walk process, i.e. $\Delta c_{t+1} = \mu + \epsilon_{t+1}$, where $\epsilon_{t+1} \sim N(0, \sigma^2)$. Derive the Sharpe ratio when $\rho_t(M_{t+1}, R_{t+1}) = -1$.

(c) **(10%)** Use the asset pricing equation and the settings in (b) to solve for r^f_{t+1}.

5. Let $X_1, X_2, ..., X_n$ be i.i.d. draws from a Poisson distribution with parameter λ and let λ have a Gamma prior with parameters α and β, i.e.

$$f(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}; \ \pi(\lambda) = \frac{1}{\Gamma(\alpha)\beta^\alpha}\lambda^{\alpha-1}e^{-\lambda/\beta}.$$

(a) **(10%)** Let $Y = \sum_{i=1}^n X_i$. Derive the posterior distribution of λ, i.e. $\pi(\lambda|y)$.

(b) **(10%)** Show that the posterior mean $E(\lambda|y)$ converges to the maximum likelihood estimator $\widehat{\lambda}_{MLE}$ as $n \longrightarrow \infty$.

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