考試科目	統計學(A) 4211	所別	金融學系金融管理組	考試時間	2 月 28 日(六)第 3 節

1. Hypothesis testing involves a set of procedures to decide whether a statistical hypothesis should be rejected.

   (a) **(10%)** Explain **in words** what p-value is and why we reject the null hypothesis when the p-value is less than the size of the test.

   (b) **(5%)** In some applications, the distribution of test statistic could be non-standard, i.e. not a known distribution. Describe how you would construct the associated probability table for statistical inference in this situation.

2. Consider a linear model $Y_i = \beta X_i + u_i$, $i = 1, 2, ..., n$ and $u_i | X_i \sim N(0, \sigma^2)$.

   (a) **(5%)** The sample moment condition $\sum_{i=1}^{n} \widehat{u}_i = 0$ might be used to estimate β. Use this moment condition to derive an estimator for the unknown parameter β.

   (b) **(5%)** Another sample moment condition $\sum_{i=1}^{n} \widehat{u}_i X_i = 0$ might also be used to estimate the model. Use this moment condition to derive another estimator for β.

   (c) **(10%)** Which one of the above is a better estimator? Prove your answer.

3. Let $X_1, X_2, ..., X_n$ denote a random sample of size $n \geq 2$ from $N(\mu, \sigma^2)$.

   (a) **(10%)** Let $\overline{X}$ be the sample average of X. Show that $n\left(\overline{X} - \mu\right)^2 / \sigma^2$ is independent of $\sum_{i=1}^{n} (X_i - \mu)^2 / \sigma^2$.

   (b) **(10%)** Let $S^2 = (1/n) \sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2$. Derive the distribution of nS^2/σ^2 and its parameter(s).

   (c) **(10%)** Let $Y_1, Y_2, ..., Y_n$ be another random sample of size n with $E(Y_i) = \mu_Y$ and $Var(Y_i) = \sigma_Y^2$. Find the constant a such that $S_Y^2 = (1/n) \sum_{i=1}^{n} \left(Y_i - \overline{Y}\right)^2 \xrightarrow{p} a$.

4. Consider a multivariate linear model $Y_i = \mathbf{X}_i' \boldsymbol{\beta} + u_i$ where $\boldsymbol{\beta}$ is a $k \times 1$ vector of coefficients, $i = 1, 2, ..., n$. The value of Y_i is either 0 or 1 and $E(u_i | \mathbf{X}_i) = 0$.

   (a) **(10%)** Show that $Var(u_i | \mathbf{X}_i)$ must be heteroskedastic, i.e., not a constant.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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(b) **(5%)** Is linear model appropriate for statistical analysis in this case? Explain.

5. Consider a random variable $X_{t+1} = \beta G_{t+1}^{-\gamma}$ and $\ln G_{t+1} = g + \epsilon_{t+1}$ where g, β and γ are all constants and $\epsilon_{t+1} \sim N(0, \sigma^2)$ is independent over time.

(a) **(10%)** Is the process for $\ln G_{t+1}$ stationary? Prove your answer.

(b) **(10%)** Let $Y_{t+1} = 1/E(X_{t+1})$. Express $\ln Y_{t+1}$ as a function of β, γ, g and σ^2.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。