國立中央大學104學年度碩士班考試入學試題

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本科考試禁用計算器

*請在答案卷（卡）內作答

參考用

Answering Problems

State with your reasoning or proofs. Please be precise and concise. No point will be graded if no explanation is provided. (答題請精準、簡捷，並皆須提示理由解釋、計算過程或證明，否則不予計分。)

1. Match the concept described in words in the upper panel with the correct symbol/value/item in the lower panel. X follows $N(\mu, \sigma^2)$ and $Z \sim N(0,1)$. K_3 and K_4 measure the skewness and kurtosis of a data set. p is the sample proportion of a specific event. (15%)

    1. Sample size need for 0.05 margin of error
    2. 100% confidence interval for p
    3. Random variable with a concave Normal quantile plot
    4. Probability of X being less than $\mu + \sigma$ in magnitude
    5. Standard deviation of Z^2
    6. About 2 for moderate sample sizes
    7. Proportion of a normal distribution that is more than 10σ from μ.
    8. Mean of $\frac{\frac{X-\mu}{\sigma}}{\sqrt{Z^2}}$
    9. Distribution of the random variable $(\mu + \sigma Z)$
    10. Probability that $Z > 1.96$ plus the probability that $Z < -1.96$

(a) 0	(b) ≈ 0	(c) 0.05	(d) 1
(e) $\sqrt{2}$	(f) 2	(g) 1/3	(h) 2/3
(i) $K_3 > 0,\ K_4 > 0$	(j) $K_3 < 0,\ K_4 > 0$	(k) 10	(l) π
(m) 100	(n) 400	(o) 900	(p) 1000
(q) $\mathbb{P}(Z < 1)$	(r) $[0,\ 1]$	(s) $N(\mu, \sigma^2)$	(t) $N(0,1)$
(u) $\chi^2(1)$	(v) $\chi^2(n-1)$	(w) $\hat{p} \pm 2 * \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	(x) $t_{0.025}(n-1)$
(y) $\hat{y} \pm 2 * \frac{s_y}{\sqrt{n}}$	(z) $[-3,\ 3]$		

2. Suppose $\{X_1,\ X_2,\ \cdots,\ X_n\}_{n=1}^{100}$ is a random sample drawn from a Poisson distribution with parameter λ. In order to test the null of $H_0 : \lambda = 0.01$ against the alternative of $H_1 : \lambda = 0.03$ with the critical region predetermined as $C = [\{X_1,\ X_2,\ \cdots,\ X_n\} | \sum_{i=1}^{100} X_i \geq 2]$. Find the probabilities of type I error, and type II error. (5%)

3. Suppose you gather a random sample $\{X_1,\ X_2,\ \cdots,\ X_n\}$ identically from an uniform distribution following

$$f(X) = \frac{1}{\theta}, \quad \text{for } 0 \leq X \leq \theta,$$
$$= 0, \quad \text{otherwise.}$$

Please use **method of moment** to estimate the unknown parameter θ. (5%)

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4. Suppose we know from a sample of data $\{(x_i, y_i)\}_{i=1}^{26}$ that their summary statistics are: $\bar{x} = 7$, $\bar{y} = 12$, $s_x^2 = 16$, $s_y^2 = 25$, and the sample correlation coefficient $r_{xy} = 0.2$. Imagine we can fit a simple linear regression model as

$$y_i = b_0 + b_1 x_i + \varepsilon_i, \quad i = 1, 2, \cdots, n.$$

Please use the above results to figure out

(1) the regression coefficients estimates b_0 and b_1 in the model. (5%)

(2) the following regression ANOVA table. (20%)

Source	df	Sum of Square	Mean Square	F stat.	R^2
Regression	1	(c)	(f)	(i)	(j)
Residual	(a)	(d)	(g)		
Total	(b)	(e)	(h)		

5. Suppose we have a dataset that contains the salary and personal information of all workers in Taiwan, and we are interested in examining the relationship between workers' education level and the amount of salary they received. An ordinary least-squares regression (OLS) model is estimated as follows:

$$Salary_i = \alpha + \beta * Education_i + \varepsilon_i,$$

where $Salary_i$ is the salary paid to work i by his or her employer; $Education_i$ is the education that work i ever received; ε_i is the residual term. A positive estimated β thus indicates a more educated worker would earn higher salary.

However, one concern in the above model is that we only are able to observe the salaries of the workers who chose to work. In other words, there are workers that are not included in our dataset because they chose not to join the workforce, and these absent workers would have received positive salary if they had chosen to work. And suppose more educated workers are less likely to quit working. If we ignore this concern and simply estimate the OLS model as above, is it Type I or Type II error that we are likely to commit, and why? (5%)

6. Suppose we need to estimate a linear regression model $Y_i = \alpha + \beta X_i + \varepsilon_i$, where residual terms ε_i is independent and identically distributed, $E(\varepsilon_i) = 0$, $Var(\varepsilon_i) = \sigma^2$, $Cov(\varepsilon_i, \varepsilon_j) = 0$ if $i \neq j$, $i = 1, \ldots, N$.

(a) Find the least square estimate of α *and* β (denoted as $\hat{\alpha}$ and $\hat{\beta}$) (5%)

(b) Find $E(\hat{\beta})$. (5%)

(c) Find $Var(\hat{\beta})$. (5%)

(d) Suppose now $Var(\varepsilon_i) = \sigma^2 \sum (X_i - \bar{X})^2$, what is $Var(\hat{\beta})$? (5%)

(e) Suppose now $\varepsilon_i = u_i / X_i$, where $E(u_i) = 0$, $Var(u_i) = \sigma^2$, $Cov(u_i, u_j) = 0$ for $i \neq j$. Is $\hat{\beta}$ still unbiased? (5%)

注意：背面有試題

7. We have a sample of 49 firms that ever announced annual earnings that are lower than analysts' forecasts. Assuming market efficiency, we expect that stock market investors are surprised by the disappointing earnings and responded with selling stocks. Suppose now we have the stock returns of these 49 firms on the announcement days, $\{r_i, i=1,\ldots,49\}$, and the standard deviation of daily returns is 7%.
   (a) How do we perform the test to support our prediction that the average announcement-day return ($\bar{r} = \sum_{i=1}^{49} r_i/49$) is negative? (5%)
   (b) If the actual population mean of announcement-day return is -1%, what is the chance of committing Type II error? (5%)

8. Assume we have the relationship between variables y and x as follows:

$$y=\beta x+e,$$

where $e \sim N(0, \sigma_e^2)$. However, we are not able to observe the "actual" x in data, but can only observe a variable $\tilde{x}$ that is related to x in the following way:

$$\tilde{x} = x + u,$$

where $u \sim N(0, \sigma_u^2)$. Suppose we regress y on $\tilde{x}$ and estimate the following model:

$$y = \gamma \tilde{x} + v$$

   (a) Show $\mathrm{Cov}(\tilde{x}, \mathrm{v}) \neq 0$. (5%)
   (b) Show the estimated $\hat{\gamma}$ is β plus an additional term ($\hat{\gamma} = \beta + c$, where c is not zero). (5%)