國立政治大學九十 五 學年度研究所 博/碩 士班入學考試

考試科目	計算機數學	所別	資訊科學8141	考試時間	3月18日 星期六 第3節

離散

一、(10%) 1. Let the language L consist of all strings of the form $a^k b^k$, where k is a positive integer. Symbolically, L is the language over the alphabet $\Sigma=\{a, b\}$ defined by

$$L = \{s \in \Sigma^* \mid s = a^k b^k, \text{ where } k \text{ is a positive integer}\}.$$

Is there a finite-state automaton that accepts L ?

(10%) 2. Find the number of bit strings of length 10 that do not contain the pattern 11.

(10%) 3. Define a sequence $a_1, a_2, a_3, \ldots$ recursively as follows:

$$a_1 = 1,$$

$$a_k = 2 \cdot a_{\lfloor k/2 \rfloor} \quad \textit{for all integers } k \geq 2.$$

Use iteration to guess an explicit formula for this sequence.

(10%) 4. Design a *mod* 3 counter i.e., an FSM (Finite State Machine) whose output at a given time equals the total number of 1s (*mod* 3) in the input stream, up to that time. As an example, here is one possible input sequence and the corresponding outputs:

(Inputs) x_n	0	1	1	0	1	1	1	1	0	1	1	0	0	1	…
(Outputs) y_n	0	1	2	2	0	1	2	0	0	1	2	2	2	0	…

(10%) 5. For each of the following four regular expressions, construct an NFA (Nondeterministic Finite Automaton) that accepts the corresponding regular language.

(a) $a^*(b+\epsilon)$

(b) $a^*b^* + c^*$.

(10%) 6. Consider the set of all one-to-one and onto functions from the set $S = \{1,2,\ldots n\}$ to itself. If f is one of these functions, a fixed point of f is an element $x \in S$ such that $f(x) = x$. If one of these functions is chosen at random, what is the probability that it has no fixed points?

備考	試題隨卷繳交
命題委員：	67 （簽章）

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線代

II、 (10%) 1. Find the general solution of the linear system whose augmented matrix has been reduced to

$$\begin{bmatrix} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

(10%) 2. Find an LU factorization of

$$A = \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}$$

(10%) 3. Diagonalize the following matrix, if possible.

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

That is, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

(10%) 4. Find the kernel of the linear transformation T from $\boldsymbol{R}^5$ to $\boldsymbol{R}^4$ given by the matrix

$$A = \begin{bmatrix} 1 & 5 & 4 & 3 & 2 \\ 1 & 6 & 6 & 6 & 6 \\ 1 & 7 & 8 & 10 & 12 \\ 1 & 6 & 6 & 7 & 8 \end{bmatrix}$$

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