| 招 生 學 年 度 | 104 | 招 生 類 別 | 碩士班 |
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| 系 所 班 別 | 應用數學系 統計碩士班 |
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| 科 目 名 稱 | 機率與統計 |
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| 注 意 事 項 | 本考科禁止使用掌上型計算機；含機率論與統計學 |
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1. (10 points) Please interpret the "Central Limit Theorem" ?

2. (5 points) Assume that X and Y are independent random variables. Can we say that X and Y are uncorrelated ? (Please verify your answer.)

3. (a) (8 points) Assume $X$ be a random variable with a symmetric probability density function $f(x)$, $-\infty < x < \infty$. Please calculate the expectation value of $X^3$ $(=E(X^3))$.

    (b) (7 points) Assume that $X \sim U(a, b)$ (a uniform random variable between $a$ and $b$). If $a < c < b$, what is the probability $P(X = c)$ ?

4. (15 points) Assume $(X_1, X_2, ..., X_n)$ be a random sample from the random variable with the probability density $f(x) = e^{-(x-\theta)}I_{(\theta,\infty)}$, $\theta > 0$. Please find the MLE (maximum likelihood estimator) about $\theta$.

5. (15 points) A random sample of 81 measurements was independently chosen from a normal population with the unknown mean ($\mu$) and the known standard deviation ($\sigma = 8$). If the sample mean ($\overline{X}$) is equal to 25, please calculate a 99% confidence interval for $\mu$. ($Z_{0.05} = 1.645$; $Z_{0.025} = 1.960$ and $Z_{0.005} = 2.575$)

6. (20 points) Assume that $(X_1, X_2, ..., X_n)$ be a random sample from $N(\mu, \sigma^2)$ where $\mu$ is unknown and $\sigma^2$ is known. Please find the UMVUE of $\mu^3$.

7. (20 points) Assume that $(X_1, X_2, ..., X_n)$ be a random sample from the random variable with the probability density $f_\theta(x) = e^{-(x-\theta)}I_{(\theta,\infty)}$, $\theta > 0$. Let $\theta_0$ and $\theta_1$ be two constants and $\theta_1 > \theta_0$. Please find a UMP (uniformly most powerful) test of size $\alpha$ for testing $H_0 : \theta = \theta_0$ v.s. $H_1 : \theta = \theta_1$.