考試科目	微積分	所別	應用數學系 911 916	考試時間	4月19日 星期六 下午第1節

1. (12%) Determine the convergence or divergence for the series

$$\sum_{k=1}^{\infty} \frac{k}{e^{k/5}}$$

2. (12%) Let $f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$,
   - a. show that f is differentiable at 0.
   - b. find $f^{(k)}(0)$, $k \geq 1$.

3. (12%) Show that the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has area πab where $a, b > 0$.

4. (12%) Let $f(x,y) = 2x^2 + 3y^2$ and C be the circular path $x^2 + y^2 = 16$, traversed counterclockwise. Evaluate the line integral

$$\oint_C \left(\nabla f \bullet \overrightarrow{N}\right) ds$$

where $\overrightarrow{N}$ is the unit normal vector to the curve C.

5. (12%) Let $f(x)$ be a continuous function on $[-1, \infty)$.
   - a. What is the average value $A(x)$ of f on $[-1, x]$ for any real number $x > -1$?
   - b. If $A(x) = \sin x$, find $f(x)$.

6. (12%) Maximize the function $f(x,y,z) = xyz$ subject to the condition

$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$$

7. (13%)
   - a. Show that the series $\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k+1}$ converges in the interval $-1 < x \leq 1$.
   - b. Show that

$$\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k+1}, \quad -1 < x \leq 1$$

8. (15%) Let S be the set of irrational numbers

$$S = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\ldots\right\}$$

That is, $a_1 = \sqrt{2}$ and for each positive integer n,

$$a_{n+1} = \sqrt{2 + a_n}.$$

   - a. Show that $a_n < 2$ for all n by induction.
   - b. Is 2 the least upper bound for S ? Explain your answer.
   - c. Find $\lim_{n\to\infty} a_n$, provided the limit exists.

備考	試題隨卷繳交