國立臺北大學 104 學年度碩士班一般入學考試試題

系（所）組別：統計學系

科　　　　目：數理統計

□可 ☑不可使用計算機

1. (30%) Let $X_1, X_2, \ldots X_n$ be iid Uniform(a, b) where $a < b$.

   a.(5%) Write down the likelihood function for the parameters a and b.

   b.(5%) Does the uniform distribution belong to exponential family?

   c.(10%) Find and show the minimal sufficient statistics for a and b.

   d.(5%) Suppose that data ={ 2.04, -0.80, 1.33, 1.25, 2.49, 0.43, 2.23, -1.80 }. Given the data, plot the region in the x-y plane where the likelihood function is non-zero. (the parameter a is plotted on the x-axis and the parameter b is plotted on the y-axis)

   e.(5%) Find the MLE of a and b for the data in (d).

2. (20%) Let a random sample of size n be taken from an uniform density on $(0,\theta)$.

   a.(5%) Let $Y_n = max\{X_1, X_2, \ldots, X_n\}$ and $Y_1 = min\{X_1, X_2, \ldots, X_n\}$. Find the conditional distribution of Y_1 given Y_n.

   b.(10%) Find the limiting distribution of $n(Y_n - \theta)$.

   c.(5%) Consider testing H_0: $\theta = 1$ v.s. H_1: $\theta = 2$. Find a test that has a significance level $\alpha = 0$.

3. (a) (15%) Compute the mean of X if the cumulative distribution function is

   (i)
   $$F(x) = \begin{pmatrix} 0 & x < 0 \\ x^2 & 0 \le x < 1 \\ 1 & 1 \le x \end{pmatrix}.$$

   (ii)
   $$F(x) = \begin{pmatrix} 0 & x < 0 \\ x^2/8 & 0 \le x < 2 \\ 1 & 2 \le x \end{pmatrix}.$$

   (b) (5 %) Prove the following statement: The probability that a random variable is within two standard deviations of its mean is at least 0.75.

   (c) (10%) Let random variables X and Y have the joint probability density function:

   $$f(x,y) = \frac{3}{2}x^2(1-|y|), \quad -1 < x < 1, \ -1 < y < 1.$$

   Calculate $\Pr(X < Y \text{ and } Y < 0)$.

   (d) (20%) Let X_1, X_2 and X_3 be i.i.d. random variables, each with probability density function

   $$f(x) = \begin{pmatrix} 0 & x \le 0 \\ e^{-x} & 0 < x < \infty \end{pmatrix}.$$

   (i) Derive the distribution of $Y = \min(X_1, X_2, X_3)$.

   (ii) Derive the distribution of $Z = X_1 + X_2 + X_3$.

   (iii) Derive the distribution of $W = X_1 / X_2$.

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