國立臺北大學 104 學年度碩士班一般入學考試試題

系（所）組別：金融與合作經營學系

科　　目：統計學

☑可 ☐不可使用計算機

一、A continuous random variable X with the probability density function:

$$f(x)=\begin{cases}\frac{1}{5}, & -2\le x\le 3\\ 0, & \textit{otherwise.}\end{cases}$$

Please derive the density function of $Y=X^2$. (10 分)

(Show the detailed derivation)

二、Assume that X follows the binomial distribution B(n, p) with:

$$f(x)=\binom{n}{x}p^x(1-p)^{n-x},\ x=0,\ 1,\ 2,\ \ldots,\ n$$

(a) Show that the moment-generating function of X is $M_X(t)=\left(1-p+pe^t\right)^n$. (3 分)

(b) Let $p=\lambda/n$. Show that X approximately follows the Poisson distribution Pois(λ) with

$f(x)=e^{-\lambda}\lambda^x/x!$, where x =1, 2, 3…, when n is sufficiently large. (7 分)

(Show the detailed derivation for each of questions)

三、Assume that X is a discrete variable with the probability mass function $f_X(x)=e^{-1}/x!$, where x =1, 2, 3…, and that Y is a continuous variable with the probability density function $f_Y(y)=e^{-y}$, where $y\ge 0$. Moreover, X and Y are independent random variables.

(a) What is the conditional probability Prob($Y\ge X\mid X=x_0$)? (5 分)

(b) What is the probability Prob($Y\ge X$)? (5 分)

(Show the detailed derivation for each of questions)

四、Let X_{it} be the random variable with $E(X_{it})=\mu_i=i$ and $Var(X_{it})=\sigma^2=1$ for i = 1, 2, .., n and t = 1, 2, …, n. Moreover, $Cov(X_{it}, X_{is})=\delta=1$ for $t\ne s$ and $Cov(X_{it}, X_{jt})=Cov(X_{it}, X_{js})=0$ for $i\ne j$. Let $Y=\left(\sum_{i=1}^{n}\sum_{t=1}^{T}X_{it}\right)\Big/(nT)$. Suppose that $n=100$ and $T=5$.

(a) What is E(Y)? (3 分)

(b) What is Var(Y)? (7 分)

(Show the detailed derivation.)

五、The price of one share of stock Z is given by S_t on the t^{th} day of the year. Assume that the differences $X_t=S_{t+1}-S_t$ appear to be independent random variables with a common distribution having mean $\mu=0$ and variance $\sigma^2=1/4$. If $S_1=100$, estimate the probability that S_{365} is

(a) What is the approximate probability of Prob($S_{365}\ge 100\mid S_1=100$)? (5 分)

(b) What is the approximate probability of Prob($S_{365}\le 0\mid S_1=100$)? (5 分)

(Give a reason for each of your answers.)

六、(15 分) A new kind of vaccine is used for two groups of patients. There are 100 patients in group A and 125 in group B. The effective numbers of group A are 59, and 68 of group B. Are there significant differences (at significant level 5%) between the effective ratios of the two groups?

七、(15 分) The following hypothesis of a survey:

$$H_0:\mu=30$$
$$H_1:\mu\ne 30$$

The standard deviation is assumed known $\sigma=5$ and the sample size is 100. With $\alpha=5\%$, computing the probability of making a Type II error if μ equals to 28.

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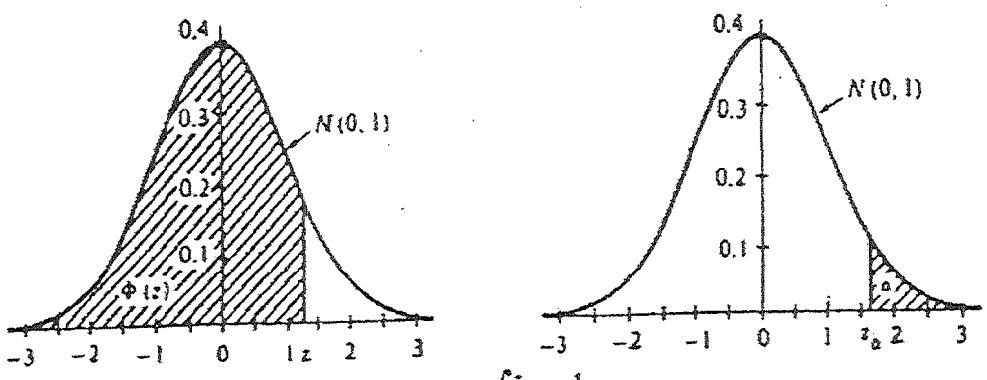
八、(20 分) A person owns his wealth W_0 at t = 0. The future daily wealth value $\tilde{W}_{t+1}$ is a random variable following $\frac{\tilde{W}_{t+1}-W_t}{W_t} \sim N(\mu,\sigma^2)$ with independent and identical distribution. He will measure the wealth after T days and concerns the probable maximum loss of his position at confidence level 1-α (The probable loss amounts are called value at risk 【VaR】). Please derive:

$$VaR = \left|\tilde{W}_T - E(\tilde{W}_T)\right| = |Z_\alpha| \times \sigma \times W_0 \times \sqrt{T}$$

The Normal Distribution



$$P(Z \le z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$[\Phi(-z) = 1 - \Phi(z)]$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

試題隨卷繳交