

考試科目	數學	所別	經濟學系	考試時間	月 星期	日 上下	午 第	節
------	----	----	------	------	---------	---------	--------	---

一、Answer the following questions (25%).

1. How is the adjoint matrix related to the cofactor matrix?
2. The product of the eigenvalues of a square matrix equals the determinant of the matrix. True or false?
3. Let  $A$  be a square matrix such that  $A^T A = I$ . What is the determinant of  $A$ ?
4. Is  $g(x) = 3x_1^2 + 2x_2^2 + x_3^2 + 4x_1x_2 + 4x_2x_3$  positive definite, negative definite or indefinite?
5. Let  $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \text{ such that } xy \geq 0 \right\}$ . Is  $V$  a vector space?

二、Solve the following problems.

1. (10%) A small, open economy with a flexible exchange rate has IS, LM, and BP curves given by

$$R = 240 - 4Y + E$$

$$R = -50 + Y$$

$$R = 105 + \frac{1}{2}Y - 2E$$

Solve, using row operations for the equilibrium interest rate  $R$ , output  $Y$ , and exchange rate  $E$ .

2. (15%) Given  $A = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$

- (a) Find the eigenvalues of  $A$ .
- (b) Diagonalize  $A$  by an orthogonal matrix.
- (c) Find the eigenvalues of  $(A^{20} + 2A)$ .

考試科目	數學	所別	經濟學系	考試時間	4月22日(上)午第 星期 下 節
------	----	----	------	------	----------------------

三, 1. (a) Find the linearization of  $f(x) = (1+x)^k$  at  $x=0$  for any number  $k$

(b) Use the approximation to estimate  $\sqrt[3]{1.009}$  (10%)

2. Evaluate

(a)  $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+10}$

(b)  $\int_0^1 \int_y^1 x^2 e^{xy} dx dy$  (15%)

3. Sketch the graph of  $y = \frac{\ln x}{x^2}$  (10%)

四, A monopolist firm sells a single product in two separate markets. Suppose that average revenue in market 1 equals  $P_1 = 20 - 2Q_1$  and average revenue in market 2 equals  $P_2 = 15 - 1.5Q_2$ . Total cost equals  $TC = 5(Q_1 + Q_2)$  ( $P_i$  represent the price in market  $i$ ,  $Q_i$  represent the quantity the firm sells in market  $i$ )

- (a) What are the first- and second-order conditions for profit maximization?  
 (b) What total quantity maximizes profit? What quantity does the firm sell in market 1 and what quantity does it sell in market 2?  
 (c) What is the price of the product for two markets? (15%)

備 考 試 題 隨 卷 繳 交

命 題 委 員:

-111-

( 簽 章 )

年

月

日